MATHEMATICAL PROGRAMMING ANALYSES OF AN ESTABLISHED TIMBERLANDS SUPPLY CHAIN WITH INTERESTS IN BIOFUEL INVESTMENTS

A Thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the
School of Chemical and Biomolecular Engineering

Georgia Institute of Technology
December 2014

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ACKNOWLEDGEMENTS

I wish to thank my thesis advisor Dr. Matthew Realff, from whom, at every meeting, I always learned something new. His continued support and advisement has truly shaped me to become an better engineer.

I am grateful for Dr. Jay H. Lee. His advisement was not only incredibly important to my research, but he also provided me with an incredible opportunity to experience working and living in the beautiful country of South Korea. I also wish to acknowledge Craig Whittaker, Ludwig Furtner, and Todd Wisecarver of Weyerhaeuser, whose assistance made this project possible. I wish to acknowledge the rest of my committee for their support and input towards my thesis.

Partial financial support of the NSF Grant GOALI program, CBET 0933392, is gratefully acknowledged. The research was also supported by the Advanced Biomass R&D Center (ABC) of Global Frontier Project funded by the Ministry of Education, Science, and Technology (ABC-2011-0031354).

Finally, I would like to thank my family and friends, who supported me throughout the ups and downs of this difficult journey.

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SUMMARY

In the push for clean and renewable fuels, timber derived biomass is a promising frontier for biofuel production in the United States. This research approaches the established timberlands biofuel implementation problem from different mathematical programming perspectives, testing feasibility and sustainability in different economic and supply related situations.

Our first study proposes that utilizing a competitive game theory approach will provide new insights into the behavior within a timberlands supply chain. We utilize Stackelberg game theory modeled with bilevel programming to represent the competing harvesting and manufacturing sectors.

In the second study, the initial bilevel model is utilized in a larger two stage multiperiod model with parameter uncertainty. In this more realistic model, the first stage contains logistical decisions around biorefinery investments, such as location and capacity, while the second stage is composed of multiple bilevel scenarios representing potential situations in the timberlands system.

Our final study focuses on long term land management strategies for the timberlands supply chain. Introduction of a new biorefinery investment means these management strategies must be adapted to continue providing consistent material flows to manufacturers as well as sustain the new production facility. A modified cyclic scheduling formulation is presented and used to model a timberlands system that can conform their planting and harvesting schedule to accommodate a new biorefinery. This cyclic model was different in that an initial startup period was added to initiate biofuel production and provide time to alter the distribution of lands.

CHAPTER I

INTRODUCTION

Recently, the United States has been facing two major issues involving energy generated from fossil fuels: a continuously increasing demand for energy and the potential impact of carbon emissions on the climate [68]. To help tackle these issues, the US government has begun to explore implementation of biofuels as a way to replace traditional fossil fuels to meet a portion of the demand for liquid fuel.

Biofuels are renewable fuels derived from biomass. They are a promising choice for sustainable transportation fuel due to the limited changes needed to existing fuel distribution and combustion technologies, particularly when converted to direct "drop-in" diesel or gasoline replacements. The first generation biofuels were derived from food crops, such as corn, soybean, sugar cane, and other oil seed crops, but the competition with using these crops as food undermines their potential as feedstocks for significant replacement of fossil fuels. The United States government has set policies that require an increase of biofuel production to 36 billion gallons per year by 2022 [25], but the United States has limited of producing first generation biofuels due to the food vs fuel issue [15]. In order to meet this government set quota, not only do many biorefineries need to be built, but next generation biofuels from nonfood biomass sources such as lignocellulosic materials need to be developed.

One potential source of lignocellulosic biomass is forestry. For the US, timber derived biofuel seems promising due to its sophisticated forestry industry. Based on data gathered in 2007, the US had about 514.2 million acres of timberlands [3]. The US generates around $200 billion per year of sales in forest products and has about one million workers in the industry. Also, before the 2008-2009 recession, the US led the world in the production and consumption of forest products; however, afterwards, they have not recovered to the same level. Fluctuations in housing markets and new technologies displacing paper products have pushed timber harvesters to look for ways to develop new, large scale, and consistent markets. Timber derived biofuels seem like a reasonable next step.

Timber biomass can be converted by many different routes to biofuels. Two classes of processes are biological and thermochemical [2]. The biological route uses chemical or mechanical pretreatment to disrupt the lignocellulosic structure and then either a chemical or enzymatic process to liberate sugars from the mixture. These sugars are recovered and subsequently fermented to fuel molecules such as ethanol. The thermochemical routes proceed through two main pathways, gasification or pyrolysis. Gasification results in a synthesis gas that can be susequently converted to medium chain length alkanes using classic Fischer Tropsch chemistry [62]. Fast pyrolysis yields bio-oils, char, and fuel gas. Bio-oils can be upgraded through refinery processes, such as hydrocracking and hydrotreating, to fuels or similarly gasified and converted via Fischer Tropsch synthesis. These conversion process can utilize residuals discarded by other facilities, possibly creating new revenue stream within the timberlands supply chain. Using these residuals for biofuel production may actually be more environmentally beneficial than discarding them through burning. Recent studies have revealed that the carbon emissions from burning wood could actually be on the same level as burning coal [34].

With the present state of the economy, however, timberlands management may be reluctant to fund large investments. Biomass conversion to biofuels is expensive, and profitability is uncertain at this point. An economic assessment of the present timberlands supply chain is important to understand the impact of introducing biorefineries to a pre-existing system. Supply chain optimization and analysis, which are becoming more prevalent within industrial settings, can provide quantitative guidance for decision makers.

1.1 Dissertation Overview

In this research, mathematical programming is used to develop supply chain models that can determine the potential impacts of biofuel production in an established timberlands supply chain.

Chapter 2 explores utilizing a Stackelberg game approach to represent the timberlands supply chain as a bilevel model. This modeling formulation places two decision making entities within the same problem; these decision makers cannot directly control the behavior of the other, but their decisions influence each other's objective. Behavioral analysis is performed on the bilevel representation by determining the influence of internal costs, retailer demands, and the future value of unharvested biomass. These results were compared with results from a single level formulation to determine the behavior that a bilevel model can reveal.

In Chapter 3, Chapter 2's bilevel timberlands model is expanded into a two stage multiperiod bilevel problem with discrete scenarios representing uncertainty. This model gives more insight on the timberlands system's decision making behavior towards biofuel production by introducing biorefinery logistics decisions and resource flow decisions through residual sales from the harvesting and manufacturing of process byproducts. This chapter will also reveal the potential difficulties of solving a two stage multiperiod model with bilevel problems as discrete scenarios.

In Chapter 4, a discrete time model for a timberlands network is developed to analyze and adapt planting and harvesting cycles to coordinate with a new biorefinery. A modified cyclic schedule formulation with an initial transition period is presented. Within this transition period, a biorefinery can be installed and harvest management strategies are evolved for biofuel production. This chapter also presents initial state generation strategies utilizing a 20 year cyclic model to simulate the established timberlands system.

1.2 Timberlands Supply Chain Literature Review

This section presents literature for research performed in timberlands and timber products supply chain modeling and optimization. Supply chain optimization is often studied by both industry and academia, as such studies can serve as useful tools for management decisions.

Philpott and Everett developed the Paper Industry Value Optimization Tool (PIVOT), which is an mixed integer linear programming (MILP) model, for use by Fletcher Challenge Paper Australasia [54]. This model determines the optimal production scheduling and supply allocation for maximization of earnings. Supply and jobs were allocated between production machineries to satisfy the demands; however, the model also tried to minimize the cost of production and machinery downtime from switching products. Bredstrom et al. developed two models for a pulp and paper supply chain in Scandinavia that set decisions over a planning horizon of 3 months [13]. The models solved scheduling problems that set varying production plans, and both models were shown to produce lower costs than manually generated solutions. The work by Weintraub and Navon in 1976 is an early analysis of the impact of transportation constraints on harvest capacity [73]. They developed an MILP to help alleviate the problem by simultaneously optimizing the silvicultural management and material transportation problems. This model determined optimal road networking for timber transportation by balancing revenue from timber sales against harvest processing costs and road infrastructure/maintenance costs. In 1986, Weintraub continued his work with Guitart and Kohn by developing a strategic timberlands tactical decision model [72]. This model, formulated as an MILP, considered large investment decisions such as building new facilities and expanding existing facilities. Similar to the previous research, optimization was performed around both harvesting practices and timber manufacturer decisions under one objective. Jones and Ohlmann developed a model to minimize costs of meeting annual demand by optimizing harvest rotations and land procurement [35]. This model optimized a discounted cost over an infinite horizon. The harvest rotation decisions were determined through forest economic modeling while the land procurement decisions were through newsvendor-type modeling [12]. In a study by Gunnarson et al., an MILP with a one year planning horizon was designed for supply decisions involving forest fuel conversion [31]. While this analysis focused on timings of production and storage of supplies, it also considered contracts with external harvesters and manufacturers.

1.3 Biorefinery Literature Review

With increasing interests in biofuel production, biorefinery research has become more prevalent. This section presents papers involving biorefinery modeling research.

With recent growth of interest in biofuels and biorefineries, a few overview papers have been written. Floudas et al. provided a more general review for studies of hybrid and single feedstock liquid fuels [26]. The paper covered biomass processes and had some discussion of optimization with uncertainty. Daoutidis et al. provided an in depth discussion on the status of biofuel research and future directions that could be pursued [20]. A review was written by Yue et al. that covers the challenges that biofuel production is currently facing [80]. The paper gives an overview of biofuel technologies and different approaches to their implementation. It also reviews many papers regarding current studies in the biofuels area.

Biorefinery research can be grouped into process design studies and supply chain studies. Design studies focus on the production facility, analyzing specification decisions such as conversion technologies, production scheduling, and capacity. Ponce-Ortega et al. proposed a disjunctive programming approach for the optimal design and configuration of biorefineries [56]. For a given product, optimal pathways were calculated, determining intermediate steps and conversion technologies based on some economic, environmental, and safety criteria. This approach was applied to a lignocellulosic biomass case study to verify its usability in real systems. Martin and Grossmann performed a study on the optimal simultaneous production of ethanol and i-butene from a diverse set of feedstock and conversion processes [44]. Their economic analysis was modeled as an mixed integer non-linear program (MINLP) and included heat integration of the production lines. It was found that i-butene production with renewable sources was feasible and that the process could generate product as a competitive diesel substitute. El-Halwagi et al. developed a multiobjective optimization model that includes economic as well as safety goals [23]. A Pareto curve was used to represent the tradeoffs between costs and safety as measured by the risks in biofuel production. Kelloway et al. [38] designed a model of a small scale biodiesel process that utilizes soybean oil and waste cooking oils [38]. The economic analysis revealed that the small scale soybean oil process was not economical, but the waste oil process had an 80% internal rate of return. It was determined that small scale waste oil processing was feasible on a local scale. Viell et al. economically analyzed two organosolv processes and proposed changes that would provide a more economically attractive production scheme [70]. They state that more improvements could be made with a better understanding of the fractionation chemistry, a more efficient and cost effective recovery system, and tighter process integration.

Biofuel supply chain research analyzes production at a larger scale. Studies can include optimal locations of multiple production facilities, resource allocation from supplier to producer, and planting decisions around different biomass types. Andersen et al. developed an MILP model for the design and planning of an integrated gasoline and ethanol supply chain, which encompassed the harvesting, production, and retail sectors [6]. Kim et al. created an MILP model governing decisions for new biorefinery investments, which included location, conversion technologies, facility capacities, and transportation logistics [40]. This study determined the financial effects of utilizing a distributed processing network versus a centralized processing network. The biomass required two conversion processes before reaching the markets. In the distributed network, these conversion processes are at separate locations, but for the centralized network, they exist in the same facilities. This study revealed that the distributed system had a higher operating cost to acquisition cost ratio than the centralized system. But, more facilities in the centralized system allowed for the flexibility of preprocessing the biomass into a more compact form of bio-oil before being transported to the major processing sites. This yielded much higher transportation costs. Sharma et al. developed a biorefinery model that optimized production decisions and maximized stakeholder value [64]. The model calculates which choice of feedstock, technology, and product provide a balance of profitability in the short term as well as value in the long term. Elia et al. developed a large scale MILP model that simulates hardwood biomass resource flows across the entire United States [24]. Dunnett et al. studied the biofuel supply chain problem through a spatial distribution approach [22]. They investigate the evolution of the supply chain with dedicated supply crops and improved conversion technologies. Alex Marvin et al. performed an economic analysis to determine the net present value of a biomass-to-ethanol supply chain in the Midwest region of the United States [45]. Their MILP determined locations and capacities of biorefineries as well as supply availability and distribution across the network. The analysis determined the economic viability of biofuels with real biomass industry data while offering economic advice for successful and profitable biofuel production. Parker et al. developed an MILP that optimizes biofuel production over various conversion technologies that compete for many different types of biomass materials [52]. Real world geographical data on the western United States were used to ensure realistic decisions on biorefinery locations.

The question that many of these studies answers is "What is the best way to setup a biofuel supply chain." The question that this thesis answers is a bit different: "What is the impact of a biorefinery on and established timberlands system." This is answered with three different points. First, established manufacturers are included within the study, meaning that the biorefinery has competition for supply. Second, the timber suppliers are modeled with more complex behavior. Finally, the timber suppliers and manufacturers are represented as separately instead of cooperating towards one overall goal.

CHAPTER II

DECISION MAKING ANALYSIS OF A BILEVEL REPRESENTATION FOR A TIMBERLANDS SUPPLY CHAIN WITH BIOFUEL PRODUCTION

This chapter details the development of a bilevel model for a timberlands supply chain with biorefinery interests. In discussions with our industrial collaborators, we learned that the timberlands system can be grouped into two major sectors: timber harvesters and timber manufacturers. Harvesting management must consider the dynamic nature of timber growth, meaning their decisions are made with regards to long term goals. Conversely, manufacturers are more concerned with the short term goal of maximizing profit from product sales. We explore how timber harvesters and manufacturers behave under these non-cooperative objectives and compare behavior with those of a system with cooperative objectives. We investigate this non-cooperative problem through the application of bilevel models and Stackelberg game theory to biorefinery investment planning in a timberlands supply chain.

2.1 Stackelberg Games and Bilevel Games

To model the harvester and manufacturer network problem, a game theoretic framework is adopted. Specifically, the formalism of a Stackelberg game will be used. Stackelberg games are an area of game theory involving two interconnected players in a turn based environment [27]. The player making the first decision is known as the leader. This decision affects the situation of the second player (follower). The follower will optimize their decisions after those of the leader are fixed. This reaction affects the leader's system. Both entities have full information of the system, so the reaction of the follower can be anticipated and influenced. In the Stackelberg game, both players' decisions have influence on one another, but they have no direct control of each other's decisions.

Stackelberg games are usually modeled with bilevel programming [9]. Bilevel programs have two levels, each with their own objective functions and constraints. The interconnectedness is represented with both sets of decision variables being present in each level. Bilevel programs and Stackelberg games have proven useful in a diverse range of supply chain and resource management research.

One of the earliest examples of bilevel programming in supply chain analysis was performed by Candler and Norton [9]. The two decision makers of the problem were the government and agricultural sectors of Mexico. By instilling policies concerning budget, production, and income, the government would attempt to maximize consumer and producer surplus. On the other hand, farmers chose to either plant food or biomass crops. Candler and Norton's results showed that the bilevel model was a plausible representation of their agriculture system. They successfully represented a turn based multi-level system with separate objective functions: one considering government policy decisions and the other considering agricultural behavior in response to those policies. Leon and Navarro formulated a resource allocation Stackelberg model involving computational resources versus energy consumption [42]. The computational providers are the leaders; they attempt to maximize their profit by reducing energy costs involved with running equipment. The availability status of the computational resources are governed by binary decision variables. The followers are the resource users, who bid on resources to fulfill their computational demands, which, in turn, affects their utility function. Through this modeling, Leon and Navarro were able to determine the bounds on energy saving for the computational provider. A case study was performed on the Liaoning province in China involving a bilevel model to optimize multi-reservoir water policies [32]. The upper level decision maker is the manager of the overall reservoir system. They must determine the policies involving the transfer to reservoirs lacking water. The lower level decision makers are the individual reservoir managers. Each manager must minimize the deviation of target water amount to the obtained amount from its own supply as well as the supply provided through transfer.

Older studies in this area required much smaller problem sizes due to the computational limitations at the time. However, with advances in computer technology and algorithms, we believe that supply chain models of a realistic scale can now be optimized with reasonable calculation times. We believe that biorefinery supply chains are a good candidate for bilevel programming analysis. Bilevel programming has been utilized in supply chain analysis before, but their use in the area of biorefinery supply chains is still relatively unexplored. Bai et al. utilized Stackelberg game theory to model resource providers and manufacturers for a biofuel supply chain as separate decision making entities [8]. They performed case studies to determine how cooperative or non-cooperative behavior effected the decisions made in the system. Wang et al. also created a bilevel model with a Stackelberg game approach that gives insights on biofuel supply chain design under a government mandate for minimum biofuel production per year [71]. It is hypothesized that analysis with a Stackelberg methodology can reveal interesting insights on how interconnected decision makers in biorefinery networks may respond to competing objectives.

2.2 Timberlands System Overview

The decision makers within the timberlands model are grouped into harvesters and manufacturers. The harvesters are represented in the leader level. The harvester problem is comprised of many different harvest zones, each varying in forest size and harvestable quantity within a time period. The manufacturers are the followers; like the harvesters, the manufacturer level has multiple facilities. Based on the amount of materials that harvesters make available, manufacturers must satisfy demands of the consumers. Each level consists of many separate providers. Each provider's goal is not to maximize their own specific profit but their level's profit as a whole. The separate entities on each level do not compete against one another, but, instead, strive to maximize a single objective. This is representative of two organizations that coordinate their individual asset management (timberlands, manufacturing facilities).

In the case of the harvester, revenue is obtained from the sale of harvested biomass. The decision to harvest in a certain zone incurs a fixed startup cost and a cost proportional to the amount harvested. Each harvesting zone has a preallocated amount of material for harvest per year. Harvest management prefers to harvest the set amount, but, if necessary, the zone manager will harvest above this quantity at an increased cost. Specific timber resources require long time periods to grow; therefore, the harvest management must consider the impact of their present decisions. In our single period formulation, this idea is reflected by assigning a value to unharvested timber that reflects on its worth in the future. Decision making at the harvester level becomes a balance between the potential present profit against the future value of the biomass material. It is considered a potential profit because, even if the material is provided, depending on flexibility, the manufacturers may not be willing to purchase it.

The manufacturer determines the optimal allocation and distribution of available supply based on transportation, acquisition, and processing costs. These decisions consider factors such as manufacturer facility capacity, retailer demands, and harvester availability. While both levels are attempting to maximize profit, they differ in overall goals. The harvesters must consider future material value in their decisions while the manufacturers attempt to maximize their present profit. In the model, the majority of manufacturing facilities are lumber mills, which produce lumber boards from timber logs. Because of limited production types, the harvesters only supply

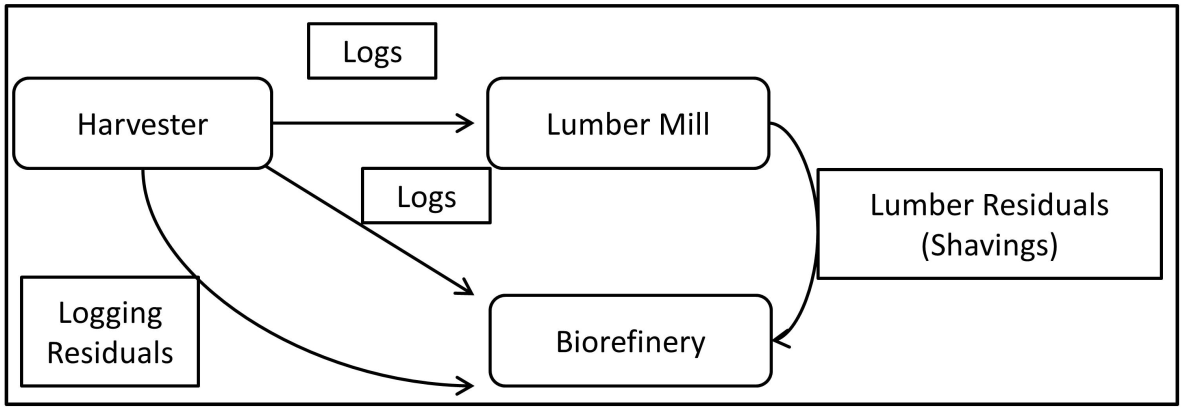

Figure 1: Timberlands System with Biorefinery

logs to the manufacturers. This lumber mill network represents a simplified version of a preexisting timberlands supply chain.

The goal of examining how existing timberland management and production might be altered by the addition of biofuel production is met by adding a biorefinery to the lumber mill network. This means that the biorefinery entity is included in the manufacturer level. The biorefinery can use logs to produce biofuel, and it can also utilize commonly discarded residuals yielded from harvesting and manufacturing processes. Thus, harvesters can also sell logging residuals to the biorefinery, obtainable from harvesting logs, and lumber mills provide the biorefinery with material through shavings as a byproduct. Figure 1 is a simple flowchart of the system.

2.3 Model Formulation

This section presents the mathematical formulation for the single and bilevel programming models developed in this research. Tables 1, 2, and 3 detail the indices, parameters, and decision variables of the timberlands supply chain model with a biorefinery.

The timberlands model captures the supply allocation behavior between timber harvesters and manufacturers. Manufacturers purchase raw materials from harvesters to satisfy retailer demands, but resource availability is limited by harvester decisions.

Table 1: Indices [Chapter 2]

i	harvest source
k	biomass type
m	manufacturer sink
l	product type
h	harvest level

Table 2: Parameters [Chapter 2]

c_{ik}	selling price of harvester i's biomass type k
o_{ik}	selling price of harvester i's biomass type k to external buyers
b_{ik}	lower bound of harvest decision x_{ik1}
f_{ik}	fixed cost of harvesting biomass type k at harvester i
r_{ikh}	processing cost per unit of biomass type k harvested at harvester i in harvest level h
q_{ikh}	upper bound of harvested quantity from harvester i of biomass type k in harvest level h
v_{ik}	amount of harvestable resources in harvester i for biomass type k
α_{kl}	conversion factor for biomass type k into product l
s_{im}	shipping cost from harvester i to manufacturer m
G_m	maximum capacity of manufacturer m
A_{ml}	maximum capacity of product l that can be produced by manufacturer m
p_{ml}	selling price from manufacturer m of product l
a_k	approximated average value of biomass type k for future value calculation
w_k	weighting of average value of biomass k for future value calculation
$\rho_{mk'}$	selling price from manufacturer m of biomass type k'
$\gamma_{kk'}$	fraction of resources of biomass type k' obtained from harvesting biomass type $k$at the harvester
$\beta_{kk'}$	fraction of resources of biomass type k' obtained from processing biomass type k at the manufacturer
$\pi_{m'mk'kl}$	binary parameter controlling flow activity of manufacturer m from manufacturer m' of material k as a byproduct of k' to produce product l
$g_{m'mk}$	price of biomass type k sold by manufacturer m' to manufacturer m
n_m	processing cost per unit of material for manufacturer m
δ	biomass pricing level, the fraction of c_{ik} charged
μ	demand level modifier, the fraction of A_{ml} developed

Table 3: Decision Variables [Chapter 2]

z_{ik}	binary variable that controls the harvester i's activity for harvesting biomass type k
x_{ikh}	fraction of harvested resources versus available harvestable resources in harvester i for biomass type k at harvest level h
y_{imkl}	purchase quantity from harvester i by manufacturer m of biomass type k for product l

Each manufacturer has their own set of products that can be made, but product type determines which forms of biomass are processable. A single level formulation of this interaction is discussed below.

2.3.1 Constraints

2.3.1.1 Harvester

$$x_{ikh}v_{ik} \leq q_{ikh}z_{ik} \ \forall \ i, k, h \tag{1}$$

Equation 1 is the upper bound on upper level decision variable x_{ikh}. The value of the upper bound is controlled by binary variable z_{ik}. The deactivation of the i^{th} harvest zone by setting $z_{ik} = 0$ prevents harvesting in inactive zones by setting the upper bound to 0 in Equation 1. x_{ikh} is the fraction of material harvested versus the amount of material k available for harvester i at harvest level h. This model has 2 material harvest levels: a preallocated level and the additional level. Timberlands management presets harvest plans over long time periods. The planned amount of harvest for a time period is governed by the preallocated level. The additional level contains material beyond the preallocated level that is harvestable at a higher cost. The costs for each level (r_{ikh}) are modeled in linear piecewise fashion. First level (preallocated) costs are consistent per ton of logs harvested, while second level (additional) costs are higher per ton of logs harvested. The higher cost of the extra level ensures that the harvester will distribute supply from the first level before proceeding to the second level. In the model, the maximum value of the preallocated level is set to 0.75, and the maximum value of the extra level is 0.25. This means the preallocated level covers the initial

75% of harvestable material and the extra level covers the remaining 25% material.

$$x_{ik1} \geq b_{ik} z_{ik} \; \forall \; i, k \tag{2}$$

The lower bound b_{ik} prevents harvesting unrealistically low amounts of material (Equation 2). The lower bound only applies to the preallocated level. In this study, b_{ik} was set to 0.5, a value that overcomes the high fixed cost of harvesting, which was set to the revenue gained from sales of 40% of the available material. This fixed cost simulates an economy of scale: as more material is harvested, the relative importance of the fixed cost declines.

2.3.1.2 Manufacturer

Equations 3, 4, and 5 are the manufacturer constraints. Constraint 3 limits manufacturer material purchases by the amount of biomass harvested.

$$\sum_m \sum_l y_{imkl} \leq v_{ik} \sum_h x_{ikh} + \sum_{k'} \gamma_{k'k} (v_{ik'} \sum_{h'} x_{ik'h'}) \; \forall \; i, k \tag{3}$$

Lower level decision variable y_{imkl} is the amount of material k demanded from harvester i by manufacturer m to product l. The first term of the right hand side of Constraint 3 is the amount of material harvested. The second term is the amount of byproduct recovered from harvesting. In this study, the byproduct yield for harvesters ($\gamma_{k'k}$) is 0.5 wet tons of logging residuals generated for each wet ton of logs harvested.

$$\sum_i \sum_k \sum_l (y_{imkl} + \sum_{m'} \sum_{k'} \pi_{m'mk'kl} \beta_{k'k} \sum_{l'} y_{im'k'l'}) \leq G_m \forall \; m \tag{4}$$

Facility capacity is also an upper bound on the manufacturer; the amount of material purchased cannot exceed the size of the facility. The second term on the lefthand side of Constraint 4 represents the amount of byproduct obtained from other manufacturers. $\pi_{m'mk'kl}$ is a binary term that governs the flow from manufacturer m' to buyer m. This term is calculated in preprocessing, and it is assumed that, when active, all materials are purchased because it represents a favorable transaction for both sides and there is enough capacity to process it.

$$\sum_k \alpha_{kl} \sum_i (y_{imkl} + \sum_{m'} \sum_{k'} \pi_{m'mk'kl} \beta_{k'k} \sum_{l'} y_{im'k'l'}) \geq \mu A_{ml} \ \forall \ m, l \qquad (5)$$

Constraint 5 covers retailer demands. The manufacturers must satisfy the demands on product, even if this means producing at a loss. Parameter μ is used as the demand level modifier (in the parametric study) with a range of 0 to 1. Manufacturers must satisfy the demands set by the demand level modifier multiplied with the maximum processing capacity. Maximum processing capacity is defined by A_{ml}. If a manufacturer has a profitable scenario with less than full processing capacity demanded, it can produce over the required amount. Any extra product is assumed to be sold at full price. For example, if $\mu = 0.5$, the manufacturer can still produce up to 80% capacity, and the extra 30% is sold at full price. Manufacturers are given greater flexibility from lower minimum demands versus situations of little flexibility due to higher demands. Simulations were run at different demand levels to determine how behavior varies in these situations. The manufacturer demand constraint also binds the harvesters; enough material must be provided to the facilities regardless of the economic situation. The byproduct yield for manufacturers (parameter $\beta_{kk'}$) is 0.3 wet tons of shavings generated for each wet ton of logs processed into lumber.

2.3.2 Objective Function

2.3.2.1 Harvester Value

Harvester revenue for the single level problem is covered in this section.

$$\text{External Revenue} = \sum_i \sum_k o_{ik} (v_{ik} \sum_h x_{ikh} + \sum_{k'} \gamma_{k'k} v_{ik'} \sum_{h'} x_{ik'h'} - \sum_m \sum_l y_{imkl}) \quad (6)$$

The difference between the amount harvested (including the byproducts) and the amount sold internally is sold at external price o_{ik} (Equation 6). In the single level problem, because the harvesters and manufacturers share an objective function, harvester revenue from internal sales is counteracted by the manufacturer purchasing costs. Therefore, internal revenue is not included in the objective function. Because timber is a long term investment, harvesters consider the value of keeping materials untouched for future sales. A value is assigned to the unharvested material with Equation 7.

$$\text{Unharvested Resource Value} = \sum_k w_k a_k \sum_i v_{ik}(1 - \sum_h x_{ikh}) \tag{7}$$

$$a_k = 0.9 i^{-1} \sum_i c_{ik} + 0.1 i^{-1} \sum_i o_{ik} \tag{8}$$

Material value a_k is calculated as the average value of the material price over all harvesters with a small influence of prices to external markets (Equation 8). This average is weighted by w_k to give the future value of biomass material k. This value is consistent across all harvesters. Equation 8 shows a 90% influence from internal prices and a 10% influence from external pricing. The i term represents the number of harvesters in the system. The average calculated for the simulations was \$41.05 per unharvested tons of logs.

2.3.2.2 Manufacturer Revenue

Manufacturer revenue is gained from product sales set at selling price p_{ml}. The product created is determined with conversion factor α_{kl} for timber materials purchased from harvesters and residuals purchased from manufacturers in the case of the biorefinery (Equation 9).

$$\text{Product Revenue} = \sum_m \sum_l p_{ml} \sum_k \alpha_{kl} \sum_i (y_{imkl} + \sum_{m'} \sum_{k'} \pi_{m'mk'kl} \beta_{k'k} \sum_{l'} y_{im'k'l'}) \tag{9}$$

$$\text{Byproduct Revenue} = \sum_m \sum_{k'} \rho_{mk'} \sum_{m'} \sum_k \sum_{l'} \pi_{mm'kk'l'} \beta_{kk'} \sum_i \sum_l y_{imkl} \tag{10}$$

Byproduct revenue is gained from sales of byproduct to the biorefinery (Equation 10). These materials are sold at $\rho_{mk'}$, the price of material sold from m for byproduct k'.

This portion of the objective function calculates the revenue from sales of byproduct for that specific manufacturer. Preprocessing for the value of $\pi_{mm'kk'l'}$ confirmed that the biorefinery was not profitable due to the high acquisition and transportation costs of this dataset when $\rho_{mk'}$ was greater than zero. However, charging only the transportation costs created a profitable situation. Therefore, in the case study, it was assumed that the costs of acquiring manufacturer byproduct materials was \$0 per ton of biomass. This assumption is reasonable because the manufacturers and biorefinery are considered to be within the same company. The variable is left within the representation in case an alternate data set is profitable for the biorefinery.

2.3.2.3 Harvester Costs

$$\text{Processing Costs} = \sum_i \sum_k (f_{ik} z_{ik} + v_{ik} \sum_h r_{ikh} x_{ikh}) \tag{11}$$

The costs of harvesting timber includes a fixed cost of processing f_{ik} that occurs with the activation of harvester i with binary variable z_{ik} (Equation 11). Furthermore, r_{ikh} is the processing cost per unit of material k within each havesting tier h. This processing cost is not uniform between each harvester.

2.3.2.4 Manufacturer Costs

$$\text{Acquisition and Transportation Costs from Another Manu.} = \sum_{m'} \sum_m \sum_k g_{m'mk} \sum_{k'} \sum_l \pi_{m'mk'kl} \beta_{k'k} \sum_i \sum_{l'} y_{im'k'l'} \tag{12}$$

$$\text{Raw Material Transportation Costs} = \sum_i \sum_m s_{im} \sum_k \sum_l y_{imkl} \tag{13}$$

$$\text{Processing Costs} = \sum_m n_m \sum_i \sum_k \sum_l (y_{imkl} + \sum_{m'} \sum_{k'} \pi_{m'mk'kl} \beta_{k'k} \sum_{l'} y_{im'k'l'}) \tag{14}$$

In Equation 12, the costs for obtaining byproduct materials from another manufacturer are priced at $g_{m'mk}$, the price of facility m purchasing from facility m' of material type k. This price is a combination of material costs and transportation costs. In our case study, however, the assumptions discussed previously for $\rho_{mk'}$ are also applied to $g_{m'mk}$, and $g_{m'mk}$ was set to include only the transportation costs. In Equation 13, the raw material transportation cost is given by parameter s_{im}, the transport cost from harvester i to manufacturer m. Processing costs are given in Equation 14 by parameter n_m, the price per ton of material processed. It is assumed that processing costs are not material dependent. The cost of purchasing materials from harvesters is not considered. While manufacturers would pay for the materials, the harvesters would gain the revenue, leaving the objective function unchanged. Therefore, it is redundant to include these transfer costs.

2.4 Bilevel Formulation

A second timberlands model was formulated as a bilevel problem to analyze the resource interactions and decision making process for biomass harvesters and manufacturers with separate objectives. The formulation utilized the same constraints and objective function terms as the single level problem as well as a few additional terms. The bilevel problem was split into an upper level controlled by the harvesters and a lower level controlled by the manufacturers. Equations 1 and 2 are harvesting constraints. Equations 6, 7, and 11 create the upper level objective function as well as the previously ignored revenue from material sales (Equation 15).

$$\text{Internal Revenue} = \sum_i \sum_k \delta c_{ik} \sum_m \sum_l y_{imkl} \tag{15}$$

Harvester internal revenue is gained from selling the lower level demands y_{imkl} at prices c_{ik}. Materials demanded are purchased at the same price by manufacturers. This is the price negotiated in contracts between the harvester and manufacturer and is assumed to be fixed. The objective of the harvester is to maximize profit by balancing revenue from the current time period and potential revenue from future sales, as well as minimize processing costs for harvesting materials. In the study, the system is analyzed at different pricing levels, defined by parameter δ. δ alters the internal biomass pricing by different percentages. Different pricing levels explore how internal pricing effects decision making at both levels.

The manufacturer constraints are Equations 3, 4, and 5. The lower level objective function has Equations 9 and 10 as revenue and 12, 13, and 14 as costs. Also, the acquisition cost of purchasing materials from harvesters is included, which is identical but of opposite sign to 15. The manufacturer objective function maximizes profit from sales while reducing acquisition and processing costs.

2.5 Bilevel Model Solution Methods with Formulation

The most common method to solve bilevel programs involves finding the optimal conditions of the lower level through KKT conversion. These KKT conditions are introduced into the top level as constraints, yielding a single level non-linear program (NLP). This added set of constraints make it so that whatever decision the top level makes, the bottom will always optimize based on the choices. The conversion of the lower level linear problem is shown below. The general form of the lower level problem (16) is converted to its KKT conditions (17). These layouts are similar to the form in [9].

$$\max_{y} f(x,y) = cx + dy, \tag{16a}$$

$$\text{subject to } Ax + By \geq b, \tag{16b}$$

$$y \geq 0 \tag{16c}$$

$$uB + vI = -d \tag{17a}$$

$$u(Ax + By - b) = 0 \tag{17b}$$

$$vy = 0 \tag{17c}$$

$$Ax + By \geq b \tag{17d}$$

$$y, u, v \geq 0 \tag{17e}$$

Equation 17a is the dual of Equation 16, determined by taking the Lagrangian of the lower level problem over the lower level decision variables. Due to the turn based timing of Stackelberg games, the leader's decisions are considered constants by the follower; during the follower's turn, the leader's decisions have already been made. In Equation 17, u is a vector of the KKT multipliers for the set of lower level constraints in section 2.3.1.2. v is a vector of the KKT multipliers for the gradient of the lower bound constraints on y ($y_{imkl} \geq 0$). I is the identity matrix.

Complementary slackness constraint 17b consists of bilinear terms, which can result in locally optimal solutions. This issue was handled by converting the complementary slackness conditions to binary constraints with a big M term (M is a large number). This converts the problem to an MILP. The conversion of the complementary slackness conditions to the big M linear representation yields Equation 18.

$$u \leq Ms \tag{18a}$$

$$v \leq Mt \tag{18b}$$

$$Ax + By - b \leq M(1 - s) \tag{18c}$$

$$y \leq M(1 - t) \tag{18d}$$

$$s, t \;\epsilon\; \{0, 1\} \tag{18e}$$

The activity of the lower level constraints is represented by the vectors of binary variables s and t. Viable M values are dependent on the parameters in the problem. In this study, the M value was set to 600. This value was determined through multiple trial runs. It represents a number high enough to not restrict the KKT constraints but low enough for reasonable calculation times. Further studies on Big M values are discussed in Chapter 3.

The flowchart in Figure 2 reviews the conversion process. Form 1 is the initial bilevel problem. The border around Form 1 is dashed to emphasize that the two levels

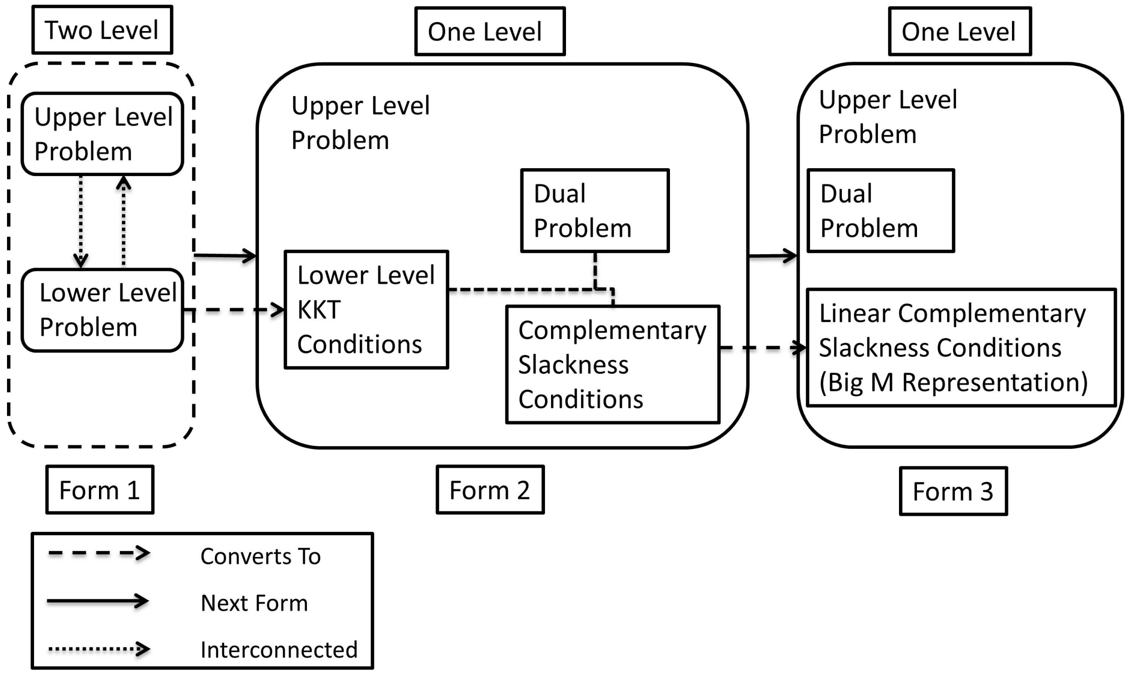

Figure 2: Bilevel Solution Method Flowchart

are separate but exist in the same problem. Equation 16 is the lower level in Form 1. The lower level problem is converted to its optimal KKT conditions (Equation 17). These KKT conditions are introduced as constraints for the upper level problem, yielding Form 2. The solid border around Form 2 represents the combination of the two levels into one. The dashed lines show that the KKT conditions can be separated into the dual problem (Equations 17a, 17d, and 17e) and the complementary slackness conditions (Equation 17b and 17c). From Form 2 to Form 3, the complementary slackness conditions are linearized through the Big M representation (Equation 18). Form 3 is optimized with an MILP algorithm.

The addition of new constraints increases the size of the problem significantly. The number of constraints that are added is equal to double the number of pre-existing lower level constraints plus the number of lower level variables. Furthermore, a new set of binary variables are introduced to the problem based on the number of constraints in the lower level.

2.5.1 Problem Specific KKT Solution

The following section displays the problem specific KKT conditions for the timberlands model. The matrix B is the gradient of the lower level constraints over variable y in the order presented in Section 2.3.1.2. Each of the $i * m * k * l$ columns in the matrix corresponds to a y variable.

$$B = \begin{bmatrix} -1 \;\; \text{rows: } i\ k \\ -1 + \sum_{m'} \sum_{k'} -\pi_{m'mk'kl}\beta_{k'k} \text{ rows: } m \\ \alpha_{kl} \;\; + \sum_{m'} \sum_{k'} \alpha_{kl}\pi_{m'mk'kl}\beta_{k'k} \text{ rows: } m\ l \end{bmatrix} \tag{19}$$

For the first set of constraints in Equation 19, there are $i * k$ rows, each corresponding to the different combination of i and k indices. As an example, assume the first row of these constraints is represented by indices $i1$ and $k1$. Given that each column represents a different combination of i, m, k,and l indices, all columns in that row that fall under $i1$ and $k1$, for all values of m and l, will have a value of -1. The other two sets of constraints must sum over indices with primes, which are iterated separate to the original unprimed indices.

$$d = \begin{bmatrix} p_{ml}\alpha_{kl} - \delta c_{ikh} - s_{im} - n_m \\ + \sum_{m'} \sum_{k'} \sum_{l'} \pi_{mm'kk'l'}\beta_{kk'}(p_{m'l'}\alpha_{k'l'} + \rho_{mk'} - g_{mm'k'} - n_{m'}) \end{bmatrix} \tag{20}$$

The term on the right hand side of Equation 17a is the gradient of the lower level objective function over y. d is a column vector with $i*m*k*l$ rows. The value in each row represents the coefficient of the row's corresponding y variable. The bottom set of terms in vector d in Equation 20 switches the primes on indices to fit with variable y_{imkl}. This switch yields a different representation of the same objective function.

2.6 Case Study Overview

In the timberlands system, the harvesters and manufacturers have separate decisions. The harvester decision variable encompasses three decisions: which harvesters are active, what types of biomass to harvest, and the amount to harvest. For the case study, the only harvested resources were logs; therefore, the only decisions of concern were harvester activity and harvest amounts (represented by the variables z_{ik} and x_{ikh} respectively). The simulated timberlands system contained 21 harvesters, which provide saw logs to the mills and produce logging residuals as byproducts.

The manufacturer has four major decisions: what to produce, what material to buy, how much to buy, and from whom to buy. The timberlands model included 7 lumber mills and 1 biorefinery, and each facility only manufactures one product. In the analyzed scenarios, processing shavings for gasoline was profitable, but the costs of obtaining logging residuals made the biorefinery produce at a loss. Therefore, the biorefinery would only buy residuals if necessary to satisfy retailer demands. Thus, the decisions of how much and who to buy from were the focus for the manufacturer analysis (represented by y_{imkl}). Mills convert logs into lumber boards and produce shavings as a byproduct. The biorefinery can utilize logs, logging residuals, and shavings to produce gasoline.

These case studies determine how these decisions change in two different models: a single decision maker or two separate decision makers. In comparing the two types of decision makers, we varied certain parameters to see how they affect the way the decisions are made. The parameters changed include biomass pricing levels (δ) and manufacturer demand requirements (μ). δ and μ are the multipliers of the respective maximum values of the parameters. Biomass pricing was varied due to its influence on the bilevel objective functions of both the harvester and manufacturer. For this study, the multiplier, δ, ranged between 0.50 to 3.00. The multiplier of the maximum demand level, μ, was set to 0, 0.10, 0.25, 0.50, 0.75, 0.90, and 1.00. While the pricing of the biomass changed in these scenarios, it was assumed that the future value utilized the static value of c_{ik} and o_{ik} with no influence from δ (Equation 8). Furthermore, these scenarios were run at future value weights (w_k) of 0.8 and 1.2.

Later, the impact of changing weights was studied from the range of 0.10 to 3.00. Uncertainty was considered when the parameters were scanned across these ranges.

Optimization was performed in GAMS (version 23.5.1). Both the single level and bilevel models were optimized with the CPLEX solver (version 12.2) on a Intel Core i7-3510QM at 2.3 GHz and 16 GB RAM. The lower level problem in the bilevel model was converted to KKT conditions with a self programmed set of functions in MATLAB R2012a. The coefficient matrices for the KKT condition constraints were fed into GAMS with a MATLAB output file.

2.7 Results

Biomass pricing analysis was performed to reveal decision making behaviors in single level and bilevel formulations of a timberlands supply chain. The specifications of the problem are shown in Table 4. Ranges of calculation times for each formulation are shown in Table 5. The continuous KKT variables cover the total number of constraints

Table 4: Model Specifications [Chapter 2]

Model Size		Variables		Constraints	
No. of Harvesters	21	Upper Level Continuous	126	Upper Level	189
No. of Manufacturers	8	Upper Level Discrete	21	Lower Level	87
Biomass Types	3	Lower Level Continuous	1008	KKT	3198
Product Types	2	KKT Continuous	1095		
Harvest Levels	2	KKT Discrete	1095		

(including $y_{imkl} \geq 0$). The discrete KKT variables are generated in the conversion of the bilinear complementary slackness constraints. For the KKT constraints, 1008 equations make up Equation 17a and 2190 equations make up Equation 18. The calculation times in Table 5 were for the initial biomass pricing level study of range 0.5 to 3.0 with several demand level values μ and a future value weighting of $w_k = 0.8$, or 80% of the approximated value. These results show that our model could be solved in a reasonable time. They also reveal that calculation times in the bilevel problem

Table 5: Ranges of Calculation Times for Biomass Pricing Analysis (secs) [future value weight $w_k = 0.8$]

Demand Level	μ	0.00	0.10	0.25	0.50	0.75	0.90	1.00
Bilevel	Min	0.150	0.634	0.676	1.037	0.401	0.560	0.262
	Max	73.481	40.212	156.773	106.426	809.314	443.072	1.349
Single Level	Min	0.043	0.051	0.063	0.051	0.050	0.056	0.054
	Max	0.134	0.185	0.233	0.238	0.232	0.241	0.234

increase when demand restrictions increase but decrease again when manufacturers lose flexibility (higher μ values). At full flexibility ($\mu = 0$), the manufacturers could refuse any unprofitable material, creating a more simple problem. The harvesters become restricted by manufacturer behavior and can only provide cheaper material. When full processing capacity is required for manufacturers, harvesters can provide the most expensive material, which yields them the most profit. Outside of these two extreme cases, the game becomes more complex due to flexibility in both levels. The single level problem showed no real trend with changing μ value, and the maximum calculation times were less than 0.25 seconds.

2.7.1 Harvester Decisions

Harvester decisions included harvester activity z_{ik} as well as amount of material harvested x_{ikh}. The first case study analyzed the system across a biomass price range of 0.5 to 2.0 across various demand levels and a future value weight of 0.8. The single level problem showed no variation in harvester activity with changing price levels. Harvesters 2, 4, 5, 6, 10 through 18, 20, and 21 harvested to full capacity while the remaining harvesters were inactive.

With a shared objective function, any revenue produced from harvester biomass sales are canceled out by acquisition costs incurred on the manufacturers, resulting in no changes in behavior over the range of prices.

Conversely, the bilevel problem showed a strong dependence on the biomass pricing

Table 6: Harvester Capacity at $\mu = 0.90$ Demand: Bilevel $[w_k = 0.8]$

Harvester		1	2	3	4	5	6	8	11	12	13	15	16	17	18	19	20	21
Cost (δ)	0.50	1	1	1	1	-	-	-	0.999	-	1	-	1	1	1	1	1	1
Level	0.75	-	-	-	-	1	-	1	-	-	0.998	-	1	1	1	1	1	1
	1.00	1	-	1	1	1	-	1	-	-	0.999	-	1	1	1	1	-	1
	1.25	1	1	1	-	1	-	-	-	-	1	-	1	1	1	1	1	1
	1.50	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
	1.75	1	1	1	1	1	1	1	-	1	1	1	1	1	1	1	1	1
	2.00	1	1	1	-	1	1	1	-	1	0.984	-	1	1	1	1	1	1

(Table 6). The bilevel behavior shows different active harvesters at each pricing step as well as decisions to harvest at less than full capacity. This more complex behavior reveals the different interactions of the bilevel formulation. Harvesters switch activity between pricing steps due to profitability thresholds being reached in the biomass pricing. This is more clearly seen when Table 6 is viewed alongside Figure 3. While active harvesters change in Table 6 between price levels 0.50 to 1.00, the amount of material harvested remains fairly static in Figure 3. As the material prices shift up, certain previously inactive harvesters will be more profitable than previously active harvesters. Therefore, harvesters will sometimes switch activity between pricing level steps.

Further analysis of these results showed that behavior in the bilevel problem could be divided into three regions. In the initial region, the pricing is low, yielding a situation where the material's future value outweighs the potential profit from immediate material sales. The harvesters only harvest enough to satisfy the contractual demands from the manufacturer but save all other material for the future. In the second region, biomass pricing proves to be more profitable in the present than in future forecasts for most harvest zones. The prices are also low enough to ensure profit for the manufacturers. In this situation of mutual gain, harvesters provide manufacturers enough material for full processing capacity. For the final region, manufacturers struggle for

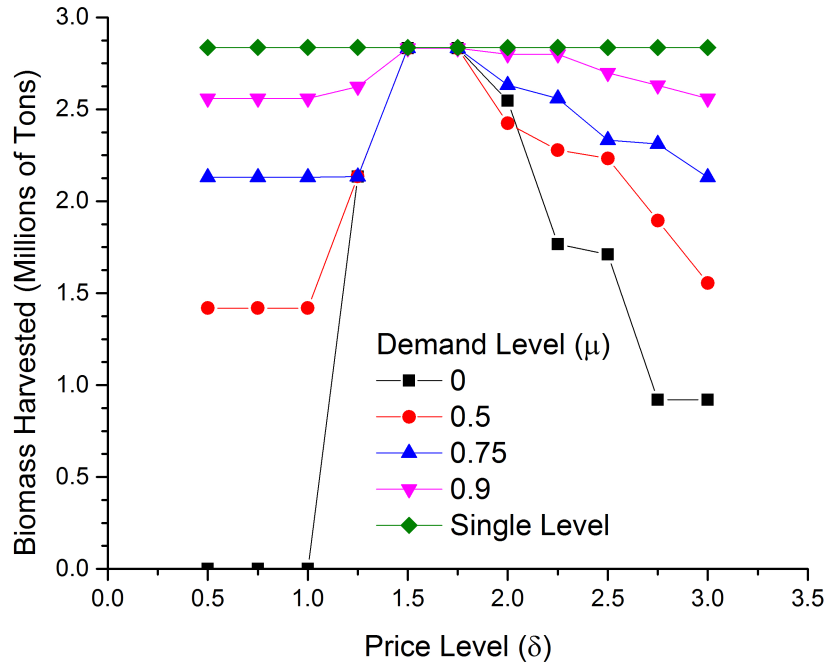

Figure 3: Tons of Biomass Harvested Versus Price Level: Bilevel $[w_k = 0.8]$

profit. As material prices increase, manufacturers are less willing to purchase material. Once all harvester zones become too costly, manufacturers only purchase enough to satisfy demands. In Figure 3, the first region exists around the 0.50 to 1.00 price level, the second region from 1.00 to 1.75, and the final region from 1.75 to 3.00. The timing of the behavior shifts is dependent on the demand. This is seen by the $\mu = 0.75$ demand results, where an increase in harvested material occurs one point later than the 0 and 0.5 demand results. These results also show that higher demands yield less drastic changes in behavior. This occurs due to the lack of flexibility in the amounts that manufacturers demand and, therefore, more freedom given to harvesters.

2.7.2 Manufacturer Decisions

The major concerns of the manufacturer level were how much material to buy and from whom. The amount purchased was dictated by the material availability as well as the economic situation.

To compare the decision making process of the manufacturer for both the formulations, the harvester decisions were set to the maximum harvest level (all $\sum_h x_{ikh} = 1$), and the minimum demand of the manufacturers was set to 0 ($\mu = 0$), giving maximum decision flexibility. Manufacturer profit in the single level model was calculated using the lower level objective function from the bilevel problem, meaning it includes material acquisition cost. This cost was ignored in the single level problem's objective function due to being canceled out by harvester revenue, but the transfer of money still occurs and must be accounted for in calculating the manufacturer profit. Table 7 shows the results of this analysis.

Table 7: Manufacturer Profit [millions of $] for Maximum Harvest [all $\sum_h x_{ikh} = 1$] and No Minimum Demand [$\mu = 0$] [$w_k = 0.8$]

Price Level (δ)	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Bilevel	240.54	213.09	185.73	158.37	131.04	103.73	77.51
Single Level	240.45	213.09	185.73	158.37	131.01	103.65	76.29

Despite static harvester decisions for both formulations, 0.50, 1.50, 1.75, and 2.00 price level scenarios showed different optimal profits. In particular, the 2.00 price level had a deviation larger than the others. With only one decision maker in the single level problem, the manufacturer can be convinced to behave non-optimally as long as the value of the whole timberlands system is optimal. In the bilevel problem, the manufacturers only consider the manufacturer level. Therefore, the bilevel manufacturer can have a higher value than the single level manufacturer. Further investigation showed different material flows between harvesters and manufacturers for these varied scenarios. Despite some differing behavior in the two formulations, the manufacturer optimal values show little variance.

The results of this section and Section 2.7.1 show that obtaining the same solution for the both formulations is difficult. Section 2.7.1 showed that harvester behavior

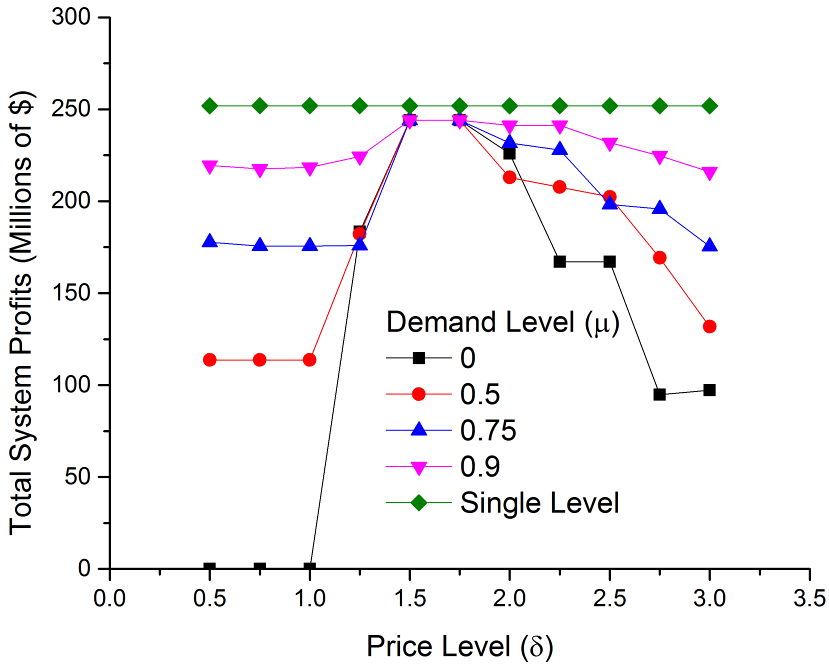

Figure 4: Total System Profits for Bilevel Model $[w = 0.8]$

changes drastically in each formulation. Very specific and possibly unrealistic restrictions must be set to incur the same behavior in both models.

2.7.3 Total System Results

The bilevel total system profits at different minimum demand requirements were compared to the static single level system profit in Figure 4. The total system profit was the sum of present profits of the harvesters and manufacturers; unharvested material value was not included. The single level system profit was never reached by the bilevel problem. The closest values vary by 3.1%. The single level has a value of \$251.97 million, and the bilevel model's highest value is \$244.07 million.

Behavior in Figure 4 is consistent with the other bilevel results. Given the flexibility, the manufacturer is capable of defending itself when prices are too high by refusing to purchase material. Also, the timberlands manager is able to postpone harvests when he believes that the future will yield more profit.

2.7.4 Effect of Increased Future Value Weighting

To confirm the consistency of the initial findings, the same demand and pricing level scenarios were simulated at a future value weighting of $w_k = 1.2$. The results are given in Figure 5, Table 8, and Table 9. Table 8 shows that the active harvesters for

Table 8: Harvester Capacity at $\mu = 0.90$ Demand: Single Level $[w_k = 1.2]$

Harvester	1	2	3	4	5	6	7	8	9	10	11
Capacity	-	1	-	1	1	1	-	-	-	1	1
Harvester	12	13	14	15	16	17	18	19	20	21	-
Capacity	1	1	1	1	1	1	1	-	1	1	-

the single level model did not change with a higher future value weight.

Table 9: Harvester Capacity at $\mu = 0.90$ Demand: Bilevel $[w_k = 1.2]$

Harvester		1	2	3	4	5	6	8	11	13	16	17	18	19	20	21
Cost Level (δ)	0.50	1	1	1	1	-	-	-	0.999	1	1	1	1	1	1	1
	0.75	-	-	-	-	1	-	1	-	0.998	1	1	1	1	1	1
	1.00	1	-	1	1	1	-	1	-	0.999	1	1	1	1	-	1
	1.25	-	-	1	-	1	-	-	-	0.976	1	1	1	1	-	1
	1.50	-	1	1	-	1	-	-	-	0.971	1	1	1	1	1	1
	1.75	1	1	1	-	1	-	-	-	1	1	1	1	1	1	1
	2.00	1	1	1	1	1	1	-	-	1	1	1	1	1	1	1

The bilevel results varied with the change in future value. As seen in Table 9, the harvesters that became active in this range of price levels differ slightly at both future value percentages. For the harvesters active in both scenarios, the the capacities and activity timing were not always consistent. The system profits for $w_k = 1.2$ showed the same rising and falling trends of the data for $w_k = 0.8$; however, the peak occurred much later and existed for a much shorter time. The higher future value of this scenario discouraged the harvester from cutting down material until a much higher price level for timber.

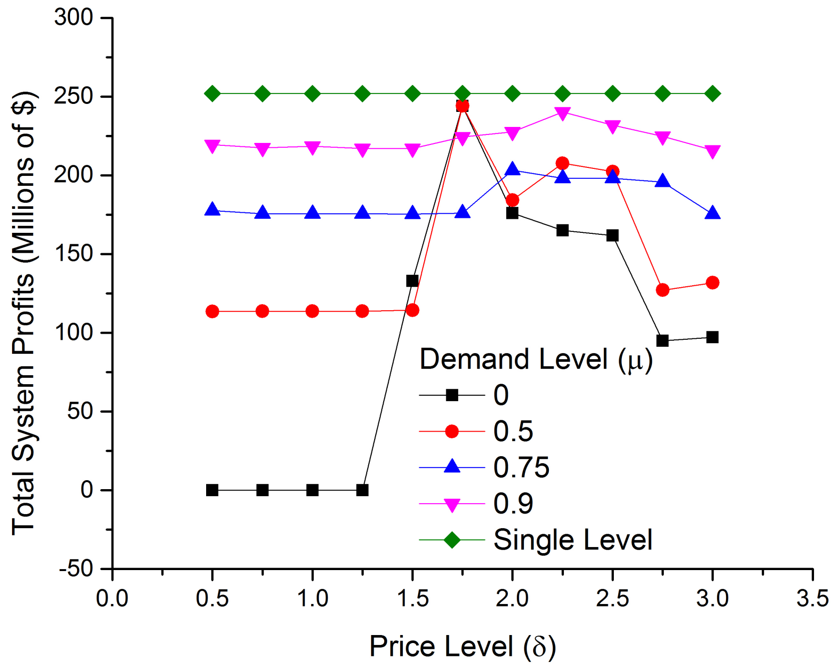

Figure 5: Total System Profits for Bilevel Model [$w_k = 1.2$]

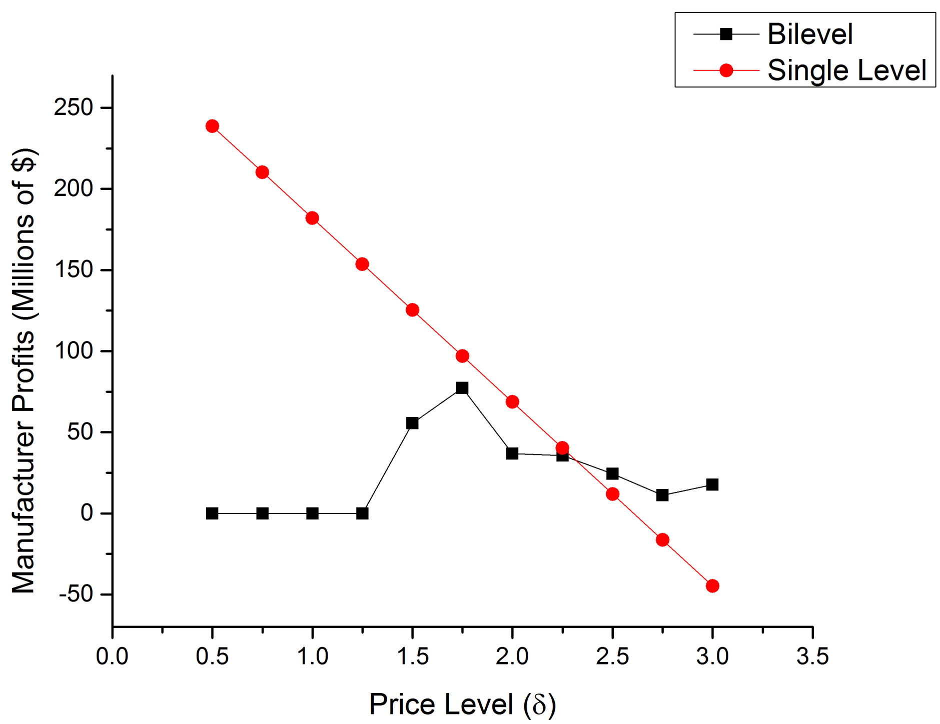

Figure 6: Manufacturer Profit at Changing Price Level [$\mu = 0$, $w_k = 1.2$]

When analyzing the harvester and manufacturer profits separately, the single level model's manufacturer level shows interesting behavior. Because of the combined profit pool of the single level objective function, the decision maker will sacrifice profitability of the manufacturer to obtain the most optimal solution of the overall system. Comparison of Figure 6 to Figure 5 confirms this and also reconfirms that the internal pricing does not affect the overall system profit in the single level formulation. In the single level problem, the manufacturer is incapable of defending itself.

The previous results show that the timber allocation decisions are strongly dictated by the state of the overall material value compared to the present potential profit. These varying decisions, in turn, also yield different situations for the manufacturers. Further analysis was performed by optimizing the timberlands model around a future value weight range of 0.1 to 3.0 with steps of 0.1. In the previous studies, when the material costs changed, there was a direct impact on the objective functions of both levels in the bilevel problem. Conversely, changes in future value only affect the harvester objective function, but changes in the harvester behavior indirectly affects the manufacturer.

In Figure 7, the single level model maintains a fairly stable present profit until the $w_k = 1.70$. Afterwards, the profit begins to drop as the harvesters become less inclined to harvest. The bilevel model follows a similar trend, but has a much sharper decline at an earlier point: between future value weight (w_k) of 0.6 and 0.7. At $w_k = 0.80$, the harvesters completely deactivate. This behavior is consistent with the changing costs analysis results in Figures 3 and 5. In Figure 3, the system is inactive at the 1.00 biomass price level, the point where the changing cost and changing future value analyses coincide. The same result can be seen in Figure 5; at biomass pricing level of 1.00, the total system profit is 0.

The results from these case studies show that imbalance between present and future value is more significant in the bilevel problem. The upper level objective

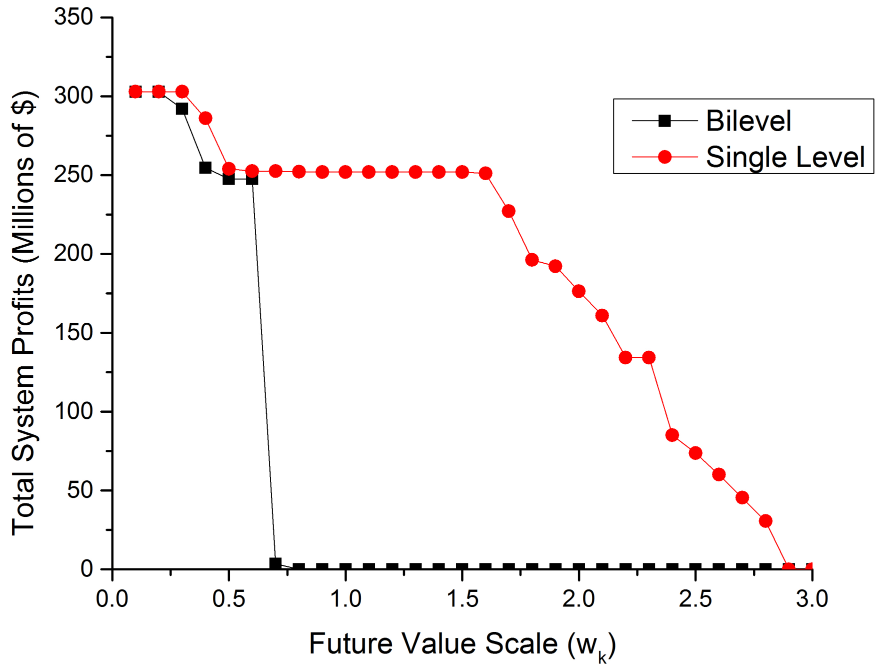

Figure 7: Total System Profit at Changing Future Value Weights [$\mu = 0$]

balances between profit from biomass sales and the value of unharvested biomass. In the single level model, the objective balances between manufacturer production revenue and unharvested future value. Furthermore, in the bilevel game, the two decision makers are non-cooperative; their objectives are to maximize their own profit. This decision making structure leads to more complex behavior in the bilevel system compared to the single level problem.

2.8 Conclusions

This results from this chapter clearly show the differences of modeling two interconnected decision makers in a timberlands supply chain either pursuing a collective objective or their own individual ones. While the single level single decision maker problem showed no changes in behavior with increasing biomass prices, the bilevel problem displayed a complex interaction between the two decision making levels in response to such variations. Specific situations yielded the same manufacturer behavior between both models, but it was shown that achieving these harvester situations in both formulations is not the default behavior. In the single level problem, the decision maker balances between revenue gained from the manufacturer and biomass material's future value, while the bilevel leader balances between biomass sales revenue and unharvested resource value. The influence of this is especially apparent in the simulations where future value was weighted differently. In the bilevel problem, the harvester is apathetic towards the situation of the manufacturer, but in the single level problem, both levels work together to maximize the total system value.

We believe that optimization data from both model formulations are useful in different areas. The bilevel model is a more accurate representation of separate decision makers and can give optimal supply allocation for situations where the value of unharvested resources is on par with the present potential profit. There is a clear need to study the coordination problem of the two systems.

2.9 *Single Level/Bilevel Comparison Simulation Data*

Chapter 2's parameter values are displayed in this section.

Table 10: Index Values [Chapter 2]

Index	Range	Value
i	$1-21$	Harvest Zones
m	1	Biorefinery
	$2-8$	Lumber Mills
k	1	Logs
	2	Logging Residuals
	3	Shavings
l	1	Lumber
	2	Gasoline
h	1	Preallocated Level
	2	Extra Level

Table 11: Biomass Selling Prices [/wet ton] [Chapter 2]

Harvester		1	2	3	4	5	6	7
Internal Price	Logs	$48.47	$50.90	$50.90	$44.70	$44.70	$44.70	$38.68
(c_{ik})	Residuals	$27.50	$27.50	$27.50	$27.50	$25.00	$25.00	$25.00
External Price	Logs	$38.78	$40.72	$40.72	$35.76	$35.76	$35.76	$30.94
(o_{ik})	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00
Harvester		8	9	10	11	12	13	14
Internal Price	Logs	$38.68	$38.68	$29.33	$29.33	$37.85	$37.85	$37.85
(c_{ik})	Residuals	$30.00	$30.00	$27.50	$27.50	$27.50	$27.50	$27.50
External Price	Logs	$30.94	$30.94	$23.46	$23.46	$30.28	$30.28	$30.28
(o_{ik})	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00
Harvester		15	16	17	18	19	20	21
Internal Price	Logs	$37.85	$43.24	$44.21	$44.21	$44.53	$46.49	$46.49
(c_{ik})	Residuals	$30.00	$30.00	$27.50	$25.00	$30.00	$25.00	$27.50
External Price	Logs	$30.28	$34.59	$35.37	$35.37	$35.62	$37.19	$37.19
(o_{ik})	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00	$8.00

Table 12: Harvesting Costs [Chapter 2]

Harvester	1	2	3	4	5	6	7
Processing Costs (r_{ikh}) [/wet ton]	\$9.60	\$10.09	\$10.09	\$8.82	\$8.82	\$8.82	\$7.62
Fixed Costs (f_{ik})	\$106.00	\$6.41	\$73.30	\$1.61	\$663.80	\$63.97	\$2,417.00
Harvester	8	9	10	11	12	13	14
Processing Costs (r_{ikh}) [/wet ton]	\$7.62	\$7.62	\$5.44	\$5.44	\$7.42	\$7.42	\$7.42
Fixed Costs (f_{ik})	\$35.16	\$1,618.06	\$355.83	\$350.29	\$226.87	\$922.14	\$159.08
Harvester	15	16	17	18	19	20	21
Processing Costs (r_{ikh}) [/wet ton]	\$7.42	\$1.94	\$1.94	\$1.94	\$2.94	\$2.94	\$2.94
Fixed Costs (f_{ik})	\$37.81	\$439.75	\$259.82	\$757.58	\$1,047.21	\$171.55	\$683.26

Table 13: Harvest Amounts [10^4 wet tons] [Chapter 2]

Harvester	1	2	3	4	5	6	7
Preallocated Level (q_{ik1})	3.645	0.21	2.4	0.06	24.75	2.385	104.145
Extra Level (q_{ik2})	1.215	0.07	0.8	0.02	8.25	0.795	34.715
Anticipated Harvest (v_{ik})	4.86	0.28	3.2	0.08	33	3.18	138.86
Harvester	8	9	10	11	12	13	14
Preallocated Level (q_{ik1})	1.515	69.72	20.22	19.905	9.99	40.605	7.005
Extra Level (q_{ik2})	0.505	23.24	6.74	6.635	3.33	13.535	2.335
Anticipated Harvest (v_{ik})	2.02	92.96	26.96	26.54	13.32	54.14	9.34
Harvester	15	16	17	18	19	20	21
Preallocated Level (q_{ik1})	1.665	16.95	9.795	28.56	39.195	6.15	24.495
Extra Level (q_{ik2})	0.555	5.65	3.265	9.52	13.065	2.05	8.165
Anticipated Harvest (v_{ik})	2.22	22.6	13.06	38.08	52.26	8.2	32.66

Table 14: Yield (α_{kl}) [tons of product/wet ton of biomass] [Chapter 2]

	Lumber	Gasoline
Logs	0.27	0.145
Residuals	-	0.14
Shavings	-	0.15

Table 15: Transportation Costs (s_{im}) [/wet ton] [Chapter 2]

Harvester	Biorefinery	Lumber Mills						
	1	2	3	4	5	6	7	8
1	$4.20	$4.81	$7.23	$9.62	-	-	-	-
2	$6.84	$5.82	$9.62	$7.68	-	-	-	-
3	$8.11	$5.82	$9.34	$6.17	-	-	-	-
4	$8.75	$6.84	$7.47	$5.47	$8.11	-	-	-
5	$7.16	$5.13	$9.39	$4.37	$9.62	-	-	-
6	$5.82	$3.62	$5.47	$5.47	$12.75	-	-	-
7	$2.61	$4.45	$4.45	$8.12	-	-	-	-
8	$4.60	$6.50	$3.50	$9.00	-	-	-	-
9	$4.51	$5.89	$3.26	$7.68	-	-	-	-
10	$7.54	$7.54	$3.90	$8.23	$14.00	-	-	-
11	$12.75	$6.38	$4.60	$4.60	$12.75	-	-	-
12	$7.79	$4.75	$4.37	$4.37	$14.00	-	-	-
13	$9.39	$6.52	$5.82	$3.62	$9.39	-	-	-
14	$14.00	$9.07	$7.47	$5.82	$7.79	-	-	-
15	$13.50	$9.07	$7.79	$5.47	$6.84	-	-	-
16	-	-	-	-	$10.11	$10.82	$10.82	$4.14
17	-	-	-	-	$10.66	$7.48	$7.48	$5.24
18	-	-	-	-	$11.45	$5.67	$5.67	$6.88
19	-	-	-	-	$6.77	$8.21	$8.21	$5.66
20	-	-	-	-	$4.75	$8.82	$8.82	$7.22
21	-	-	-	-	$6.23	$4.32	$4.32	$6.49

Table 16: Manufacturer Parameters [Chapter 2]

	Facility	Price of Shavings	Capacity	Product	100% Process Capacity	Product Price
Units		[/wet ton]	[10^6 wet tons]		[10^4 product tons]	[/product ton]
Parameter		$g_{m'mk}$	G_m		A_{ml}	p_{ml}
Biorefinery	1	-	1.00	Gasoline	14.00	$469
Lumber Mills	2	$4.20	0.52	Lumber	14.04	$504
	3	$6.84	0.46	Lumber	12.42	$550
	4	$8.11	0.46	Lumber	12.42	$588
	5	$8.75	0.51	Lumber	13.77	$450
	6	$7.16	0.32	Lumber	8.64	$462
	7	$5.82	0.30	Lumber	8.10	$490
	8	$2.61	0.27	Lumber	7.29	$520

CHAPTER III

REPRESENTATION OF BIOREFINERY INVESTMENTS IN A TIMBERLANDS SYSTEM UNDER UNCERTAINTY WITH A TWO STAGE MULTIPERIOD STOCHASTIC MODEL

This chapter extends the work of Chapter 2 by exploring bilevel problems in a multistage form for representing biofuel investment decisions. The single period model from the previous chapter is expanded to a two stage multiperiod model with parameter uncertainty. The two stage format was used to capture the timing aspect of the biorefinery investment decision, and the multiperiod form simulated uncertainty with a set of discrete scenarios with varying parameters. Uncertainty in decision making models provides decisions that consider many different possible future outcomes, creating a more realistic model that will ultimately be more useful as an analysis tool.

Uncertainty is included in many biofuel supply chain model studies. These kinds of studies are important for biofuel supply chain modeling given that biorefineries are large investments and economics are unpredictable. One of the earlier studies of uncertainty in biofuel supply chains was performed by Dal-Mas et al. [19]. Dal-Mas et al. developed an MILP model for the design of corn to ethanol supply chain for a 10 year time horizon. This model determined optimal location logistics with product price and production cost uncertainty for maximizing profit and minimizing risk case studies. For the optimal design of hydrocarbon biorefinery supply chains, Gebreslassie et al. developed a multiperiod stochastic MILP model that included demand and supply uncertainties [29]. Mansoornejad et al. designed a model with uncertainty in market scenarios and system network scenarios for a forest biorefinery supply chain

[43]. They used a stepwise methodology to determine the effect of design decisions on process operations. Sharma et al. also designed a biofuel production model with market uncertainty [63]. The model covered strategic investment decisions for the biofuels through a stochastic integer programming model. Tong et al. looked at an integrated biofuel and petroleum supply chain system [67]. The system was modeled stochastically as a two-stage MILP with scenarios involving biomass availability, fuel demand, crude oil prices, and technology evolution. It was assumed that through technology evolution, material, production, and infrastructure costs would decrease.

There have also been some studies in bilevel programming supply chain chain models with uncertainty. Capitanescu and Wehenkel determined the worst case scenarios for power flows under operational uncertainty for the determination of contingency plans [14]. Their bilevel program was solved through heuristic approximation. A traffic control problem with traffic uncertainty was studied by Chiou [18]. A min-max bilevel program was utilized where the leader managed traffic signals and the followers were the drivers. Konur and Golias tackled a truck scheduling problem with arrival time uncertainty [41]. They used a genetic algorithm approach to determine optimal time assignments for pessimistic and optimistic situations. Wogren et al. developed a model that calculates long term investments in the electricity market with regards to the investments of other companies [74]. The investing company was modeled in the upper level while the competing company decisions were considered in the lower level. Competition uncertainty was modeled through pricing variations.

Multiperiod problems with uncertainty have been used extensively in system modeling [12]. Paules IV and Floudas developed a two-stage stochastic MINLP to model a multiperiod heat-integrated distillation design problem [53]. Uncertain parameters included feed composition and flowrate, and the uncertainty was represented through discrete scenarios. Kang and Lansey used a multiperiod model to study water supply infrastructure planning [37]. They used discrete scenarios to represent uncertainties such as increased demand from population growth, changes in regulation, and public outlook. This model analyzed the situation to determine the timing of water infrastructure investments and their size. Moreno and Montagna developed a two-stage stochastic multiperiod linear generalized disjunctive programming model to study the design and planning decisions of multiperiod batch facilities with consideration to demand uncertainties [48]. The design decisions were made in the first stage before information about the product demand was known. Zhu et al. studied batch plant production in a multiperiod sense, but approached the system with a scheduling problem [82]. They modeled the system with a two-stage stochastic integer-programming model for the production of multiple products with demand uncertainties. Their model included penalties for production shortfalls and excess. Rodriguez et al. considered an inventory management problem with demand uncertainty with an MINLP [59]. The goal of this work was to propose redesign decisions for a spare parts supply chain. With the non-linear behavior in consideration, a piecewise linearization approach was used to determine the lower bound of the optimal solution. Giarola et al. developed a multiperiod MILP model to optimize an ethanol production supply chain with uncertain market conditions [30]. The supply chain was studied over an 18 year horizon divided into 6 time periods. The model considered factors such as carbon cost emissions, biomass crop management, and technology learning issues. Three case studies were considered: optimal economic and environmental situation, risk mitigation under an optimal economic situation, and risk mitigation under an optimal environmental situation. Kim et al. developed a two stage mixed integer stochastic program around a timberlands supply chain with biofuel production [39]. This study utilized discrete scenarios generated by uncertain parameters to determine the optimal estimated value of the system.

There is also research involving multiperiod bilevel games, but literature with this methodology is rarer. It seems that the combination of these two formulations is relatively unexplored. Sinha et al. modeled an oligopolistic market with a bilevel model over multiple time periods [65]. This research studied and solved the general formulation of a multiperiod multiple-leader-follower system. Su and Geunes designed a two-stage supply chain bilevel model with uncertainty [66]. They studied a supplier pricing problem where the supplier must set prices over a finite time horizon with demand uncertainties from the retailer. The bilevel model calculated the supplier's pricing decisions and the retailer's operation decisions.

We believe this combination of stochastic multiperiod programming is novel with the inclusion of a bilevel problem scenario stage. We were unable to find any recent literature that utilizes this type of representation. Analysis of solution methodologies to handle the bilevel uncertainty were performed. The research also had case studies involving economic and resource analysis of the impact of the biorefinery on our timberlands supply chain. These cases include robustness studies on the sustainability of the system.

3.1 System Overview

The system in this chapter is similar to the system discussed in Chapter 2. Chapter 2's bilevel game, formulated to model an existing timberlands system, is extended to a two stage stochastic program [12]. First stage decisions involve the biorefinery investment, and the second stage decisions are the flows of material to the biorefineries and manufacturing facilities. Harvesters provide saw logs to lumber mills and produce logging residuals as byproducts. Mills convert logs into lumber boards and produce shavings as a byproduct. The biorefinery can utilize logs, logging residuals, and shavings to produce gasoline. The first stage decision is represented through a set of binary variables corresponding to a combination of 3 locations and 3 capacities. For this study, a maximum of 1 biorefinery could be built, giving a total of 10 possible investment decisions: 9 decisions comprising of the different location/capacity combinations and the remaining decision being no investment. A diagram of the system is shown in Figure 8.

The second stage model is Chapter 2's bilevel timberlands model with harvesters as the leaders and manufacturers as the followers with each period representing a different set of uncertain parameter values. The formulation is similar to a two stage stochastic linear programming representation, but the first stage is an integer programming problem and the second stage has some additional discrete variables [60]. Harvesters provide timber material for manufacturers to process. Both levels are composed of multiple entities within the level; however, each entity's goal is not to maximize their own profit but the level's profit as a whole. This is represented by the dashed border around the two problems in Figure 8. Both levels maximize their own overall profit under different conditions. The manufacturer level has the short term goal of maximizing their present profit. In the leader level, the dynamic nature of timber resources requires the harvesters to plan over the long term. In this model, unharvested biomass is given a value that simulates potential future revenue. The harvester must find an optimal balance between harvested and unharvested resources. In this study, the system is comprised of 15 harvest zones providing logs to 4 lumber mills to produce timber.

It is assumed that the first stage biorefinery location and capacity decisions are controlled by the harvesters. This reflects a business case where the harvesters are seeking to diversify the portfolio of their outlets and maximize the efficiency of the existing biomass resources. This orientation of harvesters and manufacturers were set based on discussions with our industrial partner. The harvesters are considered upper management, and thus, should be the ones deciding on the biorefinery implementation as the leaders of the bilevel system. This placement was supported by our industrial collaborators; the timberlands is considered the core business and could make this

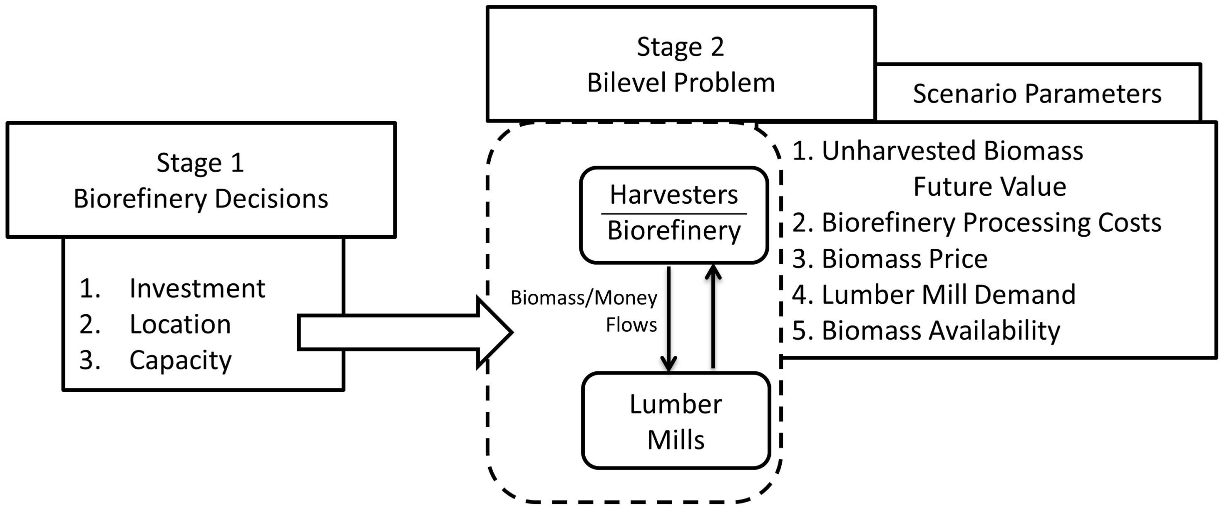

Figure 8: Two Stage Model System Diagram

large scale investment. This placement adds another set of decisions that the upper level must consider due to potential revenue from the biorefinery investment. Furthermore, the biorefinery facility creates additional dynamics in the system. Biorefineries can produce gasoline precursors from previously discarded residuals from harvesting and manufacturing processes, meaning new biomass types and resource flows are created. In a deterministic case, the first stage decisions could be combined with the harvester decisions and solved directly. However, for these more strategic decisions, parametric uncertainty may be incorporated using a multiperiod model. Each period represents a single scenario that contains a combination of uncertain parameters. The decision timeline is constructed as follows. First, the strategic decisions (the size and location of the biorefinery) are fixed using an expected value of the objective function obtained by the standard technique of weighting each scenario. However, the solution to each scenario is found by the bilevel representation of the game between harvester and manufacturer, as opposed to the more traditional set of constraints. In this study, we construct the simplex of scenarios based on taking each parameter value combination, a total of 48 scenarios (Table 17). For example, one scenario would be: [$w_{kj} = 0.7$, $\mu_j = 1.0$, $\delta_j = 1.5$, $t_j = 50\%$, and v_{ikj}&q_{ikhj} are Scen. 1]. Changing μ_j to

0.7 and keeping all other parameters the same would yield another scenario.

Table 17: Scenario Parameter Values

Future Value Weight	Bioref. Processing Cost Multiplier	Biomass Cost Multiplier	Min. Processing Demand	Biomass Availability
w_{kj}	μ_j	δ_j	t_j	v_{ikj} and q_{ikhj}
0.7	0.7	1	0	Scen. 1 (Table 45)
1.1	1	1.5	50%	Scen. 2 (Table 45)
			90%	

3.2 Model Formulation

Table 18: Indices [Chapter 3]

i	harvest source
m	manufacturer sink
k	biomass type
l	product type
h	harvest interval
u	biorefinery capacity index
e	biorefinery location index
j	scenario index

This section details the mathematical formulation developed in this chapter. Tables 18-21 detail the indices, parameters, and decision variables of the timberlands supply chain model with a biorefinery. Note that, in this chapter, the meanings of some parameters and variables have changed from Chapter 2. For example, π was a parameter in Chapter 2 but is now a variable. The parameter values are given in the Section 3.5 at the end of the chapter.

3.2.1 Upper Level Constraints

Constraints 21 and 22 are the bounds on the harvester decision x_{ikhj}. The h index represents different levels of harvest available to the timberlands managers. In this study, 2 levels exist, the preplanned level and the extra level. Harvesting from the

Table 19: Scenario Independent Parameters

c_{ik}	selling price from harvester i for biomass type k
$\hat{c}_{mk}$	selling price from manufacturer m for biomass type k
β_{ue}	annualized capital cost of biorefinery of capacity u at location e
o_{ik}	selling price of harvester i's biomass type k to external buyers
b_{ik}	lower bound of harvest decision x_{ik1}
f_{ik}	fixed cost of harvesting biomass type k at harvester i
r_{ikh}	processing cost per unit of biomass type k harvested at harvester i in harvest interval h
α_{kl}	conversion factor for biomass type k into product l
s_{im}	shipping cost from harvester i to manufacturer m
$\hat{s}_{ie}$	shipping cost from harvester i to biorefinery at location index e
G_m	maximum capacity of manufacturer m
$\hat{G}_u$	biorefinery capacity for index u
A_{ml}	maximum capacity of product l that can be produced from manufacturer m
p_{ml}	selling price from manufacturer m of product l
$\hat{p}_{lue}$	selling price of biorefinery at location e of capacity u of product l
a_k	approximated average value of biomass type k for future value calculation
$\gamma_{kk'}$	fraction of resources of biomass type k' obtained from harvesting biomass type k at the harvester
$\hat{\gamma}_{kk'}$	fraction of resources of biomass type k' obtained from processing biomass type k at the manufacturer
g_{mke}	price of biomass type k sold by manufacturer m to biorefinery at location index e
n_m	processing cost per unit of material for manufacturer m
$\hat{n}_u$	processing cost per unit of material for biorefinery of capacity index u
κ	number of biorefineries that can be introduced to the system

Table 20: Scenario Dependent Parameters

q_{ikhj}	upper bound of harvested quantity from harvester i of biomass type k in harvest interval h for scenario j
v_{ikj}	amount of harvestable resources in harvester i for biomass type k for scenario j
δ_j	biomass pricing level for scenario j
μ_j	biorefinery processing cost multiplier for scenario j
t_j	lumber mill demand level modifier for scenario j
w_{kj}	weighting of average value of biomass k for future value calculation for scenario j
ω_j	occurrence probability weight for scenario j

Table 21: Decision Variables [Chapter 3]

First Stage Variables	
D_{ue}	binary decision of biorefinery capacity index u of at location e
Second Stage Variables	
z_{ikj}	binary variable that controls the harvester i's activity for harvesting biomass type k for scenario j
x_{ikhj}	fraction of harvested resources versus available harvestable resources in harvester i for biomass type k at harvest interval h for scenario j
y_{imklj}	purchase quantity from harvester i by manufacturer m of biomass type k for product l for scenario j
π_{ikluej}	purchase quantity from harvester i of biomass type k for product l at location index e and capacity index u for scenario j
$\hat{\pi}_{mkluej}$	purchase quantity from manufacturer m of biomass type k for product l at location index e and capacity index u for scenario j

extra level incurs an additional cost. The bounds are controlled by binary variable z_{ikj}, the activity decision of a harvester. The lower bound of an active harvester (Constraint 21) is a start-up amount; this prevents the harvest of unrealistically small amounts of material. This lower bound only constrains the preplanned level of material ($h = 1$). In this study, b_{ik} has a value of 0.5, which, along with the values of the fixed costs of harvesting, were chosen to simulate an economy of scale. As the harvester provides more material, the relative importance of the fixed cost declines. The upper bound (Equation 22) is limited by the amount of available material in the scenario. The preplanned amount (q_{ikhj} when $h = 1$) is 75% of the available material (v_{ikj}), and the extra level (q_{ikhj} when $h = 2$) is the remaining 25%.

$$x_{ik1j} \geq b_{ik} z_{ikj} \;\forall\; i, k, j \tag{21}$$

$$x_{ikhj} v_{ikj} \leq q_{ikhj} z_{ikj} \;\forall\; i, k, h, j \tag{22}$$

Equations 23-26 are constraints on the biorefinery. Constraint 23 limits the number of biorefinery investments to κ. This study was limited to 1 biorefinery ($\kappa = 1$). Constraint 24 limits the number of biorefineries in a location to 1. Constraint 25 limits the amount of purchasable biomass by the amount of byproduct generated by the manufacturers. Byproduct generation of biomass type k from biomass k' is given by parameter $\hat{\gamma}_{k'k}$. In our study, $\hat{\gamma}_{k'k} = 0.3$ dry tons of shavings/wet ton of logs processed [33]. The upper limit on purchased material by the biorefinery is determined by Constraint 26. The capacity is determined by binary decision variable D_{ue}, which sets the location and size of the biorefinery. In this study, the gasoline demand on the biorefinery from external retailers is 0. This allows us to see the optimal operating conditions for biofuel production under the full decision making flexibility. This assumption is also seen in Chapter 2

$$\sum_{u}\sum_{e} D_{ue} \leq \kappa \tag{23}$$

$$\sum_{u} D_{ue} \leq 1 \; \forall \; e \tag{24}$$

$$\sum_{l}\sum_{u}\sum_{e} \hat{\pi}_{mkluej} \leq \sum_{k'} \hat{\gamma}_{k'k} \sum_{i}\sum_{l'} y_{imk'l'j} \; \forall \; m, k, j \tag{25}$$

$$\sum_{k}\sum_{l}\Big(\sum_{i} \pi_{ikluej} + \sum_{m} \hat{\pi}_{mkluej}\Big) \leq D_{ue}\hat{G}_u \; \forall \; u, e, j \tag{26}$$

3.2.2 Manufacturer Constraints

The manufacturer level is constrained by facility capacity (Constraint 27), retailer demand (Constraint 28), and material availability (Constraint 29). Retailer demand is modified by scenario parameter t_j. In Constraint 29, material availability is dependent on harvester decision x_{ikhj} as well as logging residual yield parameter $\gamma_{k'k}$. For this study, $\gamma_{k'k} = 0.3$ dry tons of residuals/wet ton of logs harvested [36]. In Chapter 2, this material flow was determined through preprocessing calculations. This model now incorporates this byproduct material flow as a decision.

$$\sum_{i}\sum_{k}\sum_{l} y_{imklj} \leq G_m \forall \; m, j \tag{27}$$

$$\sum_{k} \alpha_{kl} \sum_{i} y_{imklj} \geq t_j A_{ml} \; \forall \; m, l, j \tag{28}$$

$$\sum_{l}(\sum_{m} y_{imklj} + \sum_{u}\sum_{e} \pi_{ikluej}) \leq v_{ikj} \sum_{h} x_{ikhj} + \sum_{k'} \gamma_{k'k}(v_{ik'j} \sum_{h'} x_{ik'h'j}) \; \forall \; i, k, j \quad (29)$$

3.2.3 Objective Function

The estimated value of the upper level objective function is found by summing over all scenarios with weighting. Each scenario is multiplied to a weight parameter (ω_j), which corresponds to the scenario's probability of occurrence. In this study, all 48 scenarios of parameter values are evenly weighted, giving an occurrence probability of 0.0208.

3.2.3.1 Upper Level Value

The value of the harvester is determined by the revenue from biomass sales balanced against the unharvested resource value. A future value is assigned to simulate timberland dynamics by discouraging the harvest of all available resources. In a realistic situation, the harvest managers would save material for future time periods. Giving unharvested material a value forces the upper level decision makers to balance the benefits of harvesting and not harvesting. The future value is calculated by weighting the average present value of the biomass. The weighting is given by parameter w_{kj} while the average value a_k is calculated by Equation 33. In Equation 33, i refers to the number of harvester entities in the upper level. The average value of the material is approximated to be 90% of the average internal price and 10% of the average external price. The internal (Equation 30) and external (Equation 31) revenue also contribute to the upper level value. Internal revenue is generated from biomass sales to the biorefinery and manufacturer. External revenue is the leftover material that is sold to facilities external to the supply chain. We utilized a set pricing for the biomass instead of an equilibrium equation. An equilibrium based pricing utilizing supply-demand interactions would require a nonlinear representation. A non-linear pricing scheme would create a much more difficult set of optimal KKT conditions to solve.

With an already complex formulation, we wanted to maintain a level of simplicity to allow for a realistic sized supply chain model that is solvable within reasonable time. Our approach was to first utilize a simpler problem for initial studies and then increase the complexity of the model after becoming familiar with this formulation. The upper level objective function also includes revenue from the biorefinery (Equation 34).

$$\text{Internal Revenue for Scenario } j = \sum_i \sum_k \delta_j c_{ik} \sum_l (\sum_m y_{imklj} + \sum_u \sum_e \pi_{ikluej}) \tag{30}$$

$$\begin{aligned}\text{External Revenue for Scenario } j = &\sum_i \sum_k o_{ik}(v_{ikj} \sum_h x_{ikhj} + \sum_{k'} \gamma_{k'k} v_{ik'j} \sum_{h'} x_{ik'h'j} \\ &- \sum_l (\sum_m y_{imklj} + \sum_u \sum_e \pi_{ikluej}))\end{aligned} \tag{31}$$

$$\text{Unharvested Resource Value for Scenario } j = \sum_k w_{kj} a_k \sum_i v_{ikj}(1 - \sum_h x_{ikhj}) \tag{32}$$

$$a_k = 0.9 i^{-1} \sum_i c_{ik} + 0.1 i^{-1} \sum_i o_{ik} \tag{33}$$

$$\text{Shared Biorefinery Revenue for Scenario } j = \sum_l \sum_u \sum_e \hat{p}_{lue} \sum_k \alpha_{kl} (\sum_i \pi_{ikluej} + \sum_m \hat{\pi}_{mkleuj}) \tag{34}$$

3.2.3.2 *Manufacturer Revenue*

The manufacturer's objective function is not weighted with a probability. The manufacturer behaves by reacting to the decision of the harvester and finds the optimal solution for the situation, which is scenario dependent. Therefore, there exists an objective function for each scenario. The formulation below shows the revenue, costs, and value for each scenario j. The manufacturer's revenue is gained from product sales and byproduct sales to the biorefinery.

$$\text{Manufacturer Revenue for Scenario } j = \sum_m \sum_l p_{ml} \sum_k \alpha_{kl} \sum_i y_{imklj} \tag{35}$$

$$\text{Byproduct Revenue for Scenario } j = \sum_m \sum_k \hat{c}_{mk} \sum_l \sum_u \sum_e \hat{\pi}_{mkluej} \tag{36}$$

3.2.3.3 *Upper Level Costs*

The only costs to harvesters are the processing costs (Equation 37). f_{ik} is the harvesting startup cost while r_{ikh} is the cost of harvesting from the extra level of material. r_{ikh} is 0 for $h = 1$. The biorefinery investments incur all other costs in the objective function. These costs include an annualized capital cost for the investment (Equation 38), the biomass acquisition costs from harvesters and manufacturers (Equations 39 and 40), transportation costs from harvesters (Equation 41), and processing costs (Equation 42). The acquisition costs of biomass from manufacturers (g_{mke} in Equation 12) is a combination of the price to purchase the biomass and the transportation costs.

$$\text{Processing Costs for Scenario } j = \sum_i \sum_k (f_{ik} z_{ikj} + v_{ikj} \sum_h r_{ikh} x_{ikhj}) \tag{37}$$

$$\text{Annualized Capital Cost for All Scenarios} = \sum_u \sum_e \beta_{ue} D_{ue} \tag{38}$$

$$\text{Acquisition Costs from Harvester for Scenario } j = \sum_i \sum_k \delta c_{ikj} \sum_l (\sum_u \sum_e \pi_{ikluej}) \tag{39}$$

$$\text{Acquisition Costs from Manu. for Scenario } j = \sum_m \sum_k \sum_e g_{mke} \sum_l \sum_u \hat{\pi}_{mkluej} \tag{40}$$

$$\text{Biorefinery Transportation Costs from Harvesters for Scenario } j = \sum_i \sum_e \hat{s}_{ie} \sum_k \sum_l \sum_u \pi_{ikluej} \tag{41}$$

$$\text{Biorefinery Processing Costs for Scenario } j = \mu_j \sum_u \hat{n}_u \sum_e \sum_k \sum_l (\sum_i \pi_{ikluej} + \sum_m \hat{\pi}_{mkluej}) \tag{42}$$

3.2.3.4 Manufacturer Costs

Similar to the biorefinery, the manufacturer costs involve acquisition (Equation 43), transportation (Equation 44), and processing costs (Equation 45).

$$\text{Acquisition Costs from Harvester for Scenario } j = \sum_i \sum_k \delta c_{ikj} \sum_l \sum_m y_{imklj} \quad (43)$$

$$\text{Transportation Costs for Scenario } j = \sum_i \sum_m s_{im} \sum_k \sum_l y_{imklj} \quad (44)$$

$$\text{Processing Costs for Scenario } j = \sum_m n_m \sum_i \sum_k \sum_l y_{imklj} \quad (45)$$

A few further assumptions are made regarding the system model. First, it is assumed that, by contractual obligation, sufficient material must be provided from the harvester so that the manufacturer can satisfy its minimum final demand requirements. This also ensures that the harvester cannot drive the problem to an infeasible state. Second, the demand constraint on manufacturers is only a lower bound. Any products manufactured in excess are sold at full price. Furthermore, all purchased materials are manufactured into product. Any harvested material left unpurchased is sold to buyers external to the system for a lower price.

3.3 Results

Optimization was performed in GAMS (version 23.5.1) using the CPLEX solver (version 12.2) on an Intel Core i7-3510QM at 2.3 GHz and 16 GB RAM. The lower level problem in the bilevel model was converted to KKT conditions with a self-programmed set of functions in MATLAB R2012a. The KKT conditions were fed into GAMS with an output file from MATLAB.

3.3.1 Benchmark and Big M Test

Benchmark tests were run for a smaller two stage problem with 8 scenarios. The 8 scenarios were generated by varying the future value weight, biomass pricing, and biomass availability parameters. Several tests were performed using different big M values, and the results are shown in Table 22. While methods have been found to determine the maximum value necessary for the Big M parameter [69], the structure of our problem was different. To solve our bilevel problem, we convert bilinear KKT complementary condition constraints to a pair of Big M integer constraints. While the upper bound of the original constraint can be easily found, the issue lies with the KKT multiplier. Within the context of our problem, there is no upper bound on this variable, meaning a maximum value of M cannot truly be found.

The results show calculation times for solving the two stage problem with enumeration through every biorefinery decision. While calculation time varies little between M values of 300 to 500, an overall trend of longer calculation time for larger M values can be seen. This indicates the importance of setting the right M value across different problem constraints to ensure reasonable computation time and also optimal solutions. Each set of lower level constraints had M values specific to its grouping. For example, the M values for the set of lumber mill capacity constraints were different than the set for biomass availability. These values were approximated by determining the maximum possible value of the left hand side of the associated KKT constraint and rounding upwards. The optimal solution for the set of M values was compared against a higher reference to ensure consistency. The reference M value was set by steadily increasing the M values until the optimal value became consistent and the solution was reasonable. Values that were too large resulted in longer calculation times and would sometimes cause non-discrete values for the binary variables. The non-discrete results are due to the solver converging on values very close to 0 and 1 [7].

Table 22: Benchmark Results for Enumeration

M Values	Varied	200	300	400	500	600
Total Calculation Times (s)	45.15	57.85	70.82	69.71	70.60	88.43

M values were adjusted and the problem was solved until the correct solution could no longer be obtained with the reduction of an M value. M values set too low would result in infeasible or inconsistent solutions due to poorly restricted constraints in the KKT conditions. In this problem, M values were set for the lumber mill capacities (M=100), retailer demands (M=50), biomass availability (M=200), and non-negativity constraint for the lower level decision variable (M=200). The reference M was set to 500. Attempts were made to solve the full multistage problem without enumeration; however, calculation times were over 8 hours for the varied M values. The solver algorithms may not be able to efficiently handle solving multiple bilevel problems simultaneously. This suggests some form of decomposition algorithm that is less naive than the complete enumeration used here.

3.3.2 Timberlands System Results

The full 48 scenario problem was solved by enumerating through all scenarios and all 10 binary decision choices, giving 480 separate problems. Enumeration through the 480 problems required 132.06 seconds of calculation time.

The average time for each problem was 0.275 seconds, and the times ranged from 0.057 to 1.145 seconds. Attempts were made to solve problems by enumerating through only the biorefinery decisions, creating 10 problems in total (instead of 480 problems). The large number of scenarios in these problems (48) led to unreasonable calculation times.

Table 23: Optimal Values of Each Biorefinery Decision [$ millions]

	NB	Loc. 1	Loc. 2	Loc. 3
NB	177.89	-	-	-
small	-	183.13	189.58	186.08
medium	-	175.89	179.34	173.91
large	-	166.48	172.42	162.09

The optimal biorefinery decision for our full two-stage problem was a small capacity facility at location 2 (Table 23). This decision gave an optimal solution value of \$189.58 million. Overall, the small biorefineries were able to generate a value increase to the system. For medium sized facilities, placement in location 2 was the only option for a more profitable system. Medium biorefineries in locations 1 and 3 as well as all large biorefineries had a lower expected value than a system with no biofuel production.

The expected profits from the harvesters are given in Table 24. The total expected profit yielded by harvesters was \$101.77 million. The expected value from unharvested biomass was \$82.21 million. An average of 2.60 million wet tons of logs were harvested, and an average of 2.15 million wet tons of logs were saved for the future. 54.7% of the available trees were expected to be harvested. This means adding the biorefinery did not overly strain the timber supply. Furthermore, almost all of the harvested logs were sent to lumber mills for processing, with only an average of 0.633 million wet tons sold to external markets. This shows that the harvester does not over harvest its materials.

Table 24: Harvester Expected Profits with Small Biorefinery at Location 2 [\$ millions]

Harvester	1	2	3	4	5
Profit	1.03	0.18	1.59	0.04	20.71
Harvester	6	7	8	9	10
Profit	1.23	26.06	6.90	7.89	0.67
Harvester	11	12	13	14	15
Profit	1.59	6.11	26.50	0.77	0.41

These results can be compared to Table 25, the optimal allocation of a system without a biorefinery. Harvesters 10 through 12 are inactive. The amount of logs harvested (1.467 million tons) is 43.61% less than the system with the small biorefinery. Without biofuel production, the value of each unit of biomass harvested decreases significantly. A major source of revenue came from converting the low value residual and shavings byproducts to gasoline. This pushed the harvester to provide more materials.

Table 25: Harvester Expected Profits with No Biorefinery [$ millions] (harvesters 10-12 inactive)

Harvester	1	2	3	4	5	6
Value	0.77	0.18	1.66	0.04	21.22	0.44
Harvester	7	8	9	13	14	15
Value	11.91	6.08	2.08	18.60	0.47	0.15

For the optimal biorefinery system, the lumber mill results are shown in Table 26. The lumber mills are extremely profitable and produce at near full capacity. When compared to a system with no biorefinery (Table 27), the profitability of selling byproduct to the biorefinery is shown to push the lumber mills to higher production, yielding an average of 49.7% increase in profit for each mill.

Table 26: Lumber Mill Results with Small Biorefinery in Location 2

Lumber Mill	1	2	3	4
Expected Profit ($ millions)	42.02	28.80	31.33	36.66
Expected Processing Capacity	100%	100%	99.8%	100%

Table 27: Lumber Mill Results with No Biorefinery

Lumber Mill	1	2	3	4
Expected Profit ($ millions)	28.12	18.12	20.84	25.92
Expected Processing Capacity	74.4%	73.3%	73.3%	77.0%

The biorefinery provides an average $6.48 million profit. Production utilizes an average of 0.78 million dry tons of residuals and 0.591 million dry tons of shavings. The facility runs at an average processing capacity of 78.34%, yielding an average of 130.41 million gallons of gasoline. Analysis of all scenarios determined that the lowest processing capacity for the biorefinery was 67.18%. Even in the worst situations, the biorefinery still produces at a realistic level. The highest capacity reached was 99.93%. From the sales of biomass byproduct, harvesters generate $25.75 million and manufacturers generate $5.91 million. By comparison, when sold to the external

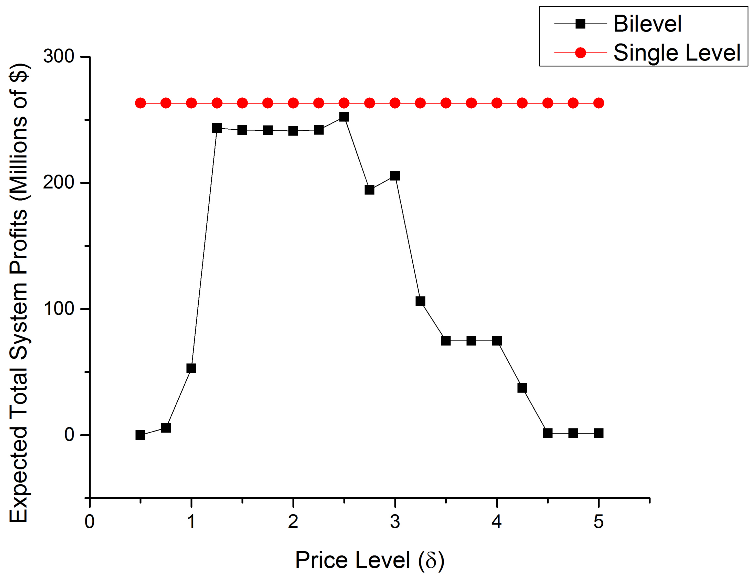

Figure 9: Varied Biomass Cost Results

market, the logging residuals would generate \$6.24 million and the shavings would generate \$4.73 million. Not only is the biorefinery profitable, but it generates additional revenue for the harvesters and manufacturers. Furthermore, the inclusions of gasoline production has no detrimental effects on the established timberlands supply chain. This can be further seen in the lumber mill results in Table 26. The results show that, even with a biorefinery, manufacturers run at full or near full capacity.

To determine the utility of bilevel models as second stage scenarios, a simplified two stage model was compared with a single stage model. For the single stage model, the objective and constraints are a combination of the two levels in the bilevel problem. The two stage formulation was simplified by removing the uncertainty in demands and biomass costs, leaving only 8 scenarios in the second stage. These two models were compared by optimizing over a range of biomass costs: $\delta = 0.5$ to 5.0 with intervals of 0.25. The results are shown in Figure 9.

While the single level solution showed no variance in behavior with changing biomass cost, the bilevel model's decision making was more complex. The behavior follows similarly to the results from Chapter 2. With lower biomass costs, harvesters prefer to save material. As prices become more favorable to the harvester, material is provided to the manufacturer for production. As the prices become too high, the biorefinery no longer profits. This system differs in the inclusion of a biorefinery decision and also by the biorefinery being located in the top level. From $\delta = 1.25$ to 3.00, the optimal solution for the bilevel problem was a small biorefinery in location 2. For all other ranges, the optimal solution was no biorefinery. For the single level problem, the optimal solution was a small biorefinery in location 2. This result shows how bilevel scenarios can differentiate from single level scenarios. It also shows the importance of the multiperiod format; under a deterministic model, a decision could be made that may be optimal in that specific situation, but not for the range of uncertainties in the system.

It should be noted that these results were specific to the configuration of this case study. Given a different set of parameters, it is very possible for another biorefinery decision to be more optimal. With a completely different set of uncertain parameters, the solution could vary. For example, if all biomass availability is incredibly high, a larger biorefinery could yield a more optimal solution. These possibilities are made more apparent in the next section, where not every scenario's optimal decision was consistent with the overall problem's results.

3.3.3 Scenario Studies

Optimal solutions were also calculated for each scenario. For the results of the 48 scenarios, only 2 decisions were chosen: 11 scenarios were optimal for no biorefinery (NB) while the remaining 37 were optimal for a small facility at location 2. The average value of the optimal solutions was \$192.35 million with a range of \$104.42 million to \$278.60 million. The scenario configurations for the worst and best optimal value solutions are shown in Table 28. The results of the scenario specific study

Table 28: Worst and Best Value Scenarios

Scenario Parameter	Future Value Weight	Processing Cost Multiplier	Biomass Cost Multiplier	Min. Processing Demand	Biomass Availability
	w_{kj}	μ_j	δ_j	t_j	q_{ikhj} and v_{ikj}
worst	0.7	1	1	90%	Scen. 1
best	1.1	0.7	1.5	any	Scen. 2

showed that the minimum processing demand had very little effect on the best value solutions but was important in distinguishing the worst value solution. In the most profitable scenarios, lumber mills already process at full capacity, rendering the demand constraint irrelevant. This means the lower bound on demand is inactive. The results also showed that a higher biomass cost led to a better solution. This result is discussed with Table 29.

The 11 scenarios with optimal solution of no biorefinery were further analyzed. Minimum processing demand t_j had a smaller influence on the biorefinery investment decision. 5 scenarios were 0% demand, 4 were 50% demand, and 2 were 90% demand. The settings of the other parameters were more strict: 9 of the scenarios had higher future value weight (w_{kj}), 9 of the scenarios had lower biomass cost (δ_j), 10 of the scenarios had higher processing costs (μ_j), and 8 scenarios chose availability scenario 1 (v_{ikj} and q_{ikhj}).

Further analysis was performed to determine the effect of the scenario parameters on the optimal scenario solution. The 48 scenarios were compared and grouped. The optimal solutions were grouped based on the scenario parameter being studied. Because each scenario parameter has only 2 values (other than lumber mill demand t_j), the scenario solutions can be paired based on the parameter being analyzed; for any scenario in this set, a partner scenario can be found where the system is the same except for the studied parameter. The differences for these pairs were found for each scenario parameter. For the case of lumber mill demand, the solutions were grouped in sets of 3. The results are shown in Table 29.

Table 29: Scenario Parameter Impact on Optimal Solution (Scenario 2 - Scenario 1) [millions $]

Modifies	Future Value	Availability	Bioref. Processing	Biomass Cost	Min. Processing Demand		
Parameter	w_{kj}	q_{ikhj} & v_{ikj}	μ_j	δ_j	t_j		
Scenario 1	0.7	Scen. 1	0.7	1	0%	50%	0%
Scenario 2	1.1	Scen. 2	1	1.5	50%	90%	90%
Average Diff.	36.14	67.39	-26.43	37.23	-0.44	-0.44	-0.88
Max. Diff.	50.59	57.61	-34.62	35.43	-2.03	-1.73	-3.76
Min. Diff.	22.99	75.06	-20.83	39.45	0.00	0.00	0.00
Standard Dev.	9.18	7.29	5.94	1.17	0.68	0.63	1.31

It can be seen that the change in t_j had minimal effect on the optimal solution. Overall, biomass availability had the largest impact. Future value and biomass cost were second largest, and processing costs were next. Standard deviation calculations showed that biomass costs and lumber mill demand had the most consistent effects on the optimal solutions. Higher biomass costs led to a higher profit value for the harvester level. While the initial assumption towards this result was due to more revenue being made with higher biomass value, the amount of material harvested is actually lower in these scenarios. Table 30 shows the average changes with changing δ_j value (the biomass cost multiplier). Despite the lower harvest, the lumber mill production slightly increases (0.0373%), with the largest deviation being 1.1653%. The biorefinery has a large decrease in production due to less residuals being generated. It can also be seen that the decisions to harvest less material deducts more from the external biomass sales than internal.

Table 30: Differences in Biomass Amounts/Facility Capacities with Changing Biomass Prices ($[\delta = 1.5] - [\delta = 1.0]$)

	Millions of Tons of Logs		Processing Capacities	
	Harvested	Sold Externally	Lumber Mills	Biorefinery
Average Difference	-0.2626	-0.2633	0.0373%	-4.4882%
Largest Deviation	0.3920	-0.9274	1.6153%	-15.9737%

In the case of the higher biomass costs, less biomass flow creates a more profitable objective. Table 31 shows the differences in profit and value of the upper level problem. The decrease in biorefinery profit and increase in unharvested material total value is consistent with the decrease in material flow, but the harvester profits and overall objective function increase in value. This behavior is consistent with results seen in Chapter 2. Changing biomass prices creates a situation where a different configuration of active harvesters can yield a better result. This is due to each harvester being unique in amount of available material, harvesting cost, and biomass pricing. Furthermore, higher biomass costs increase the value of each unit sold to the lumber mills. The very slight increase in lumber mill processing shows that the increased biomass prices are not high enough to discourage buying. Therefore, the increase in biomass revenue and unharvested material value could outweigh the losses incurred in the biorefinery from the lower harvests.

Table 31: Differences in Profit/Value with Changing Biomass Prices ($[\delta = 1.5] - [\delta = 1.0]$)

[$ millions]

	Profit		Value	
	Biorefinery	Harvester	Unharvested Material	Objective Function
Average Difference	-11.947	42.862	7.473	38.388

3.3.4 Activity Studies

Studies were performed to see how system behavior would change in a situation where certain harvesters and manufacturers were unavailable. The results for inactive harvesters are shown in Table 32. Unharvested material provided no value to the objective function for the harvesters that were inactive. The optimal biorefinery investment decision did not change due to one inactive harvester; no matter which harvester was shut down, a small biorefinery at location 2 was still the optimal choice.

One inactive harvester did not have a significant enough effect to discourage a biorefinery investment. The largest decreases in expected value occurred when zones with large biomass amounts were disabled (e.g. Harvesters 7, 9, and 13); the total unharvested timber value is much larger in these zones. The original optimal objective value was $189.58 million.

Table 32: Expected Value with Inactive Harvesters [$ millions]

Harvester	1	2	3	4	5
Expected Objective Value	187.90	189.37	187.85	189.52	167.33
Harvester	6	7	8	9	10
Expected Objective Value	188.08	141.86	183.52	158.44	177.70
Harvester	11	12	13	14	15
Expected Objective Value	178.29	180.78	155.74	185.72	188.34

The results for inactive manufacturers are shown in Table 33. A single inactive lumber mill would shift the system to no longer invest in biofuel production. With an inactive manufacturer, a system without a biorefinery is expected to generate much higher profit than one with a smaller biorefinery in location 2 (the most optimal decision when biofuel investment is required). These results show that the interaction of the manufacturer on the system has a significant impact on the timberlands system and biofuel production. Demand from manufacturers increases the harvesting activity in the system. Also, the additional supply of shavings byproduct assists in the economic viability of the biorefinery.

Table 33: Expected Value with Inactive Lumber Mills [$ millions]

Inactive Lumber Mill	1	2	3	4
Optimal Value w/o Biorefinery	173.057	176.481	175.267	172.054
Most Optimal Value w/ Biorefinery	146.889	167.828	159.388	145.535
Most Optimal Biorefinery Decision	Small Loc. 2	Small Loc. 2	Small Loc. 2	Small Loc. 2

Two sets of serial deletion studies were run across the set of harvesters. The harvesters were sorted by impact on the estimated value. The first study deactivated harvesters in the order of smallest impact (Table 34). The Inactive Harvester Series column details the order of deactivation; moving down a row deactivates an additional harvester.

Table 34: Serial Deletion: Smallest Impact Series

Inactive Harvester Series	Expected Value [$ millions]									
	NB	S1	S2	S3	M1	M2	M3	L1	L2	L3
4	177.85	183.08	189.52	186.03	175.84	179.28	173.86	166.43	172.36	162.04
2	177.66	182.88	189.32	185.81	175.63	179.08	173.64	166.22	172.15	161.82
15	176.51	181.62	188.09	184.53	174.35	177.81	172.35	164.90	170.83	160.50
6	175.11	180.00	186.59	182.96	172.74	176.28	170.78	163.27	169.28	158.91
1	173.47	178.43	184.98	181.41	171.15	174.67	169.22	161.67	167.64	157.34
3	171.91	176.71	183.23	179.68	169.41	172.88	167.48	159.89	165.80	155.61
14	168.10	172.53	179.40	175.23	165.17	168.92	162.93	155.55	161.71	151.07
8	162.55	166.36	173.12	169.22	158.79	162.48	156.75	149.01	155.07	144.76
12	153.49	156.67	163.26	159.23	148.75	152.27	146.55	138.90	144.86	134.51
11	142.25	145.04	151.60	147.77	136.96	140.30	135.06	126.74	132.24	122.92
10	130.32	132.27	138.56	134.69	124.04	126.93	121.86	113.59	118.43	109.57
5	107.63	109.03	115.30	110.73	100.52	103.55	97.87	89.81	94.75	85.43
9	76.35	77.13	82.67	79.24	68.19	70.72	66.30	57.11	61.33	53.58

The study determined that the system can deactivate up to 13 harvesters and still function. Deactivating another harvester pushes the system into an infeasible region; the 90% processing capacity constraints on the manufacturer can no longer be satisfied. In the serial deletion studies, removing harvesters showed an overall trend in decreasing difference between the optimal estimated value and the estimated value of a system without a biorefinery. These two values never converge due to reaching problem infeasibility.

In the second serial deletion study, harvesters were deactivated starting from the largest impact on the expected value. Whenever an infeasibility was encountered, that harvester was reactivated and the harvester with the next largest impact was deactivated. These results also showed a decreasing trend in difference between the optimal estimated value and the estimated value with no biorefinery. The smallest difference occurred when the last possible harvester was deactivated with a value of $2.98 million.

Table 35: Serial Deletion: Largest Impact Series

Harvesters		Expected Value [$ millions]									
Added	Removed	NB	S1	S2	S3	M1	M2	M3	L1	L2	L3
-	7	131.30	134.71	141.86	136.35	126.47	130.60	123.65	116.11	122.58	111.30
-	13	99.75	99.08	106.87	101.41	90.30	95.14	88.48	79.57	86.53	75.93
-	9	-	-	-	-	Infeasible	-	-	-	-	-
9	5	76.07	72.73	80.56	73.95	63.58	68.55	60.92	52.45	59.18	48.12
-	10	-	-	-	-	Infeasible	-	-	-	-	-
10	11	-	-	-	-	Infeasible	-	-	-	-	-
11	12	-	-	-	-	Infeasible	-	-	-	-	-
12	8	69.22	65.07	73.05	66.34	55.88	60.94	53.32	44.58	51.35	40.44
-	14	-	-	-	-	Infeasible	-	-	-	-	-
14	3	67.25	62.71	70.63	64.00	53.44	58.46	50.95	42.07	48.78	38.02
-	1	-	-	-	-	Infeasible	-	-	-	-	-
1	6	-	-	-	-	Infeasible	-	-	-	-	-
6	15	65.98	61.08	68.97	62.32	51.75	56.78	49.27	40.33	46.99	36.30
-	2	-	-	-	-	Infeasible	-	-	-	-	-
2	4	65.93	61.02	68.91	62.26	51.69	56.71	49.21	40.26	46.93	36.23

3.4 Conclusions

This chapter explored the development of a two stage multiperiod bilevel model to represent biorefinery investments in an established timberlands system under uncertainty. The calculation times and result validity were reliant on the correct selection of big M values. Enumerating the biorefinery decisions through all scenarios was shown to be much faster than solving the whole two stage problem using the standard branch-and-bound algorithm. Applying the optimization model to a particular case study demonstrated that such a model can indeed provide a useful initial analysis of timber management implementing biofuels into its supply chain. The results showed that 1 biorefinery could coexist with the established lumber mills without detrimental effect on the original supply flows. The optimal solution showed that the biorefinery could be profitable as well as provide additional revenue to the system through byproduct purchases. It was shown that the bilevel scenarios and multiperiod format provided a more complete view of the model compared to the more limited scope of single level and deterministic models. Further analysis was performed to determine the influence of the scenario parameters on the system behavior. Finally, system activity studies were performed by deactivating harvesters and lumber mills. It was discovered that the biorefinery was very dependent on the byproduct generated from all 4 lumber mills. Serial deletion studies showed that the system could still operate with 13 out of 15 harvesters deactivated. Also, no matter which harvesters were inactive, a small biorefinery at location 2 was always a more profitable choice than no biorefinery. Further developments of this model include increasing the supply chain size, which would allow for investment of more than 1 biorefinery facility. The increase in size and complexity would lead to a more difficult problem to solve. Alternative solution methods may be required to accommodate for these improvements. We believe that this research has led to a useful decision analysis tool for timberlands management that are considering the development of biofuels.

3.5 Two Stage Multiperiod Simulation Data

Chapter 3's parameter values are displayed in this section.

Table 36: Timberlands System Index Values [Chapter 3]

Index	Range	Value
i	$1-15$	Harvest Zones
m	$1-4$	Lumber Mills
k	1	Logs
	2	Logging Residuals
	3	Shavings
l	1	Lumber
	2	Gasoline
h	1	Preallocated Level
	2	Extra Level
u	Small, Medium, Large	Biorefinery Capacity
e	Loc. 1, Loc.2, Loc. 3	Biorefinery Location
j	$1-48$	Scenario

Table 37: Biomass Selling Prices [/wet ton] [Chapter 3]

Harvester ($i =$)		1	2	3	4	5
Internal Price (c_{ik})	Logs	$48.47	$50.90	$50.90	$44.70	$44.70
	Residuals	$27.50	$27.50	$27.50	$27.50	$25.00
External Price (o_{ik})	Logs	$38.78	$40.72	$40.72	$35.76	$35.76
	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00
Harvester ($i =$)		6	7	8	9	10
Internal Price (c_{ik})	Logs	$44.70	$38.68	$38.68	$38.68	$29.33
	Residuals	$25.00	$25.00	$30.00	$30.00	$27.50
External Price (o_{ik})	Logs	$35.76	$30.94	$30.94	$30.94	$23.46
	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00
Harvester ($i =$)		11	12	13	14	15
Internal Price (c_{ik})	Logs	$29.33	$37.85	$37.85	$37.85	$37.85
	Residuals	$27.50	$27.50	$27.50	$27.50	$30.00
External Price (o_{ik})	Logs	$23.46	$30.28	$30.28	$30.28	$30.28
	Residuals	$8.00	$8.00	$8.00	$8.00	$8.00

Table 38: Harvesting Costs ($k = logs$, $h = 2$) [Chapter 3]

Harvester ($i =$)	1	2	3	4	5
Processing Costs (r_{ikh}) [/wet ton]	$9.60	$10.09	$10.09	$8.82	$8.82
Fixed Costs (f_{ik})	$106.00	$6.41	$73.30	$1.61	$663.80
Harvester ($i =$)	6	7	8	9	10
Processing Costs (r_{ikh}) [/wet ton]	$8.82	$7.62	$7.62	$7.62	$5.44
Fixed Costs (f_{ik})	$63.97	$2,417.00	$35.16	$1,618.06	$355.8
Harvester ($i =$)	11	12	13	14	15
Processing Costs (r_{ikh}) [/wet ton]	$5.44	$7.42	$7.42	$7.42	$7.42
Fixed Costs (f_{ik})	$350.29	$226.87	$922.14	$159.08	$37.81

Table 39: Yield (α_{kl}) (logs:wet tons/others:dry tons) [Chapter 3]

	Lumber [ton/ton]	Gasoline [gallons/ton]
Logs	0.27	47.75
Residuals	-	92.21
Shavings	-	98.80

Table 40: Transportation Costs (s_{im}) [/ton] [Chapter 3]

Harvester (i)	Lumber Mills (m)			
	1	2	3	4
1	$4.20	$4.81	$7.23	$9.62
2	$6.84	$5.82	$9.62	$7.68
3	$8.11	$5.82	$9.34	$6.17
4	$8.75	$6.84	$7.47	$5.47
5	$7.16	$5.13	$9.39	$4.37
6	$5.82	$3.62	$5.47	$5.47
7	$2.61	$4.45	$4.45	$8.12
8	$4.60	$6.50	$3.50	$9.00
9	$4.51	$5.89	$3.26	$7.68
10	$7.54	$7.54	$3.90	$8.23
11	$12.75	$6.38	$4.60	$4.60
12	$7.79	$4.75	$4.37	$4.37
13	$9.39	$6.52	$5.82	$3.62
14	$14.00	$9.07	$7.47	$5.82
15	$13.50	$9.07	$7.79	$5.47

Table 41: Lumber Mill Parameters ($k = logs$, $l = lumber$) [Chapter 3]

		Shavings Price	Capacity	100% Process Capacity	Lumber Price	Processing Cost
Units		[/dry ton]	[10^6 wet tons]	[10^4 product tons]	[/product ton]	[$/ton]
Parameter		$\hat{c}_{mk}$	G_m	A_{ml}	p_{ml}	n_m
Lumber Mills (m)	1	$10.00	0.53	14.31	$588	$29
	2	$10.00	0.52	14.04	$504	$28
	3	$10.00	0.46	12.42	$550	$32
	4	$10.00	0.46	12.42	$588	$29

Table 42: Transportation Costs from Harvester to Biorefinery ($\hat{s}_{ie}$) [/ton of biomass] [Chapter 3]

Harvesters	Loc. 1	Loc. 2	Loc. 3
1	$9.52	$10.82	$10.11
2	$7.47	$7.48	$10.66
3	$6.96	$5.67	$11.45
4	$7.31	$8.21	$6.77
5	$7.88	$8.82	$4.75
6	$3.45	$4.32	$6.23
7	$5.37	$8.13	$4.64
8	$9.55	$10.58	$12.95
9	$8.28	$6.27	$13.58
10	$9.71	$6.27	$8.99
11	$5.04	$2.35	$8.74
12	$8.58	$6.88	$7.22
13	$7.48	$9.15	$12.16
14	$11.77	$13.03	$5.37
15	$7.48	$10.19	$8.61

Table 43: Cost of Acquisition for Biorefinery from Facilities: Biomass Price and Transportation Cost Combined (g_{mke}) [/ton of biomass] [Chapter 3]

Lumber Mill	Loc. 1	Loc. 2	Loc. 3
1	$22.75	$18.13	$15.04
2	$23.25	$20.58	$18.58
3	$19.52	$16.27	$17.48
4	$17.47	$16.27	$21.77

Table 44: Biorefinery Parameters ($l = gasoline$) [Chapter 3]

Facility Size	Capacity	Product Price $\hat{p}_{lue}$ [/gallon]			Processing Cost	Annualized Capital Cost β_{ue} [$ millions]		
Index u	$\hat{G}_u$ [tons]	Loc. 1	Loc. 2	Loc. 3	$\hat{n}_u$ [/ton]	Loc. 1	Loc. 2	Loc. 3
Small	175	$1.454	$1.485	$1.488	$73.20	$61.737	$59.885	$64.207
Medium	250	$1.458	$1.464	$1.448	$67	$77.448	$75.877	$79.090
Large	350	$1.424	$1.442	$1.403	$57.77	$94.274	$92.389	$96.160

Table 45: Biomass Availability Scenarios (v_{ikj}) [10^4 wet tons of logs] [Chapter 3]

	Scenario	
Harvester	1	2
1	4.86	3.43
2	0.28	0.52
3	3.20	4.47
4	0.08	0.14
5	33.00	67.17
6	3.18	4.05
7	138.86	122.52
8	2.02	24.78
9	92.96	77.48
10	26.96	39.77
11	26.54	36.70
12	13.32	31.46
13	54.14	111.66
14	9.34	11.54
15	2.22	4.22

CHAPTER IV

MODIFIED CYCLIC SCHEDULING REPRESENTATION TO COORDINATE FOREST LAND MANAGEMENT WITH BIOFUEL PRODUCTION

The studies in Chapters 2 and 3 utilized bilevel models to represent the interactions between the separate harvester and manufacturer decision making sectors. While interesting behavior results were exhibited, these models were limited by their single period form. While the value of unharvested resources in future time periods was represented, it was a simplistic approximation that did not give a complete view of the timberland's behavior. This chapter will tackle this issue by focusing on the scheduling problem for harvesting and planting cycles coordinated with investment decisions for biorefinery infrastructure. The scheduling problem is represented with a cyclical model that is preceded by a transition period. In this transition period, the harvest management can install a biorefinery and modify their forestry schedule to better provide material for biofuel production.

A cyclical programming approach was used to represent a 20 year time period in a timberlands supply chain. Cyclical models are often used to simulate systems where product demand is consistent over long time periods [77]. In a cyclical model, the values of the final state variables must be within a range of the values of the initial state variables, creating a consistent cycle. Cyclical programming is a common approach used in chemical engineering process scheduling problems [55, 61, 75, 16]. It has also been seen in supply chain research [58, 78, 46]. We believe cyclical scheduling was a reasonable approach for a timberlands system because it simulated consistent planting/harvesting cycles. Furthermore, the duration of 20 years was chosen because it is the approximate age range of loblolly pine, the timber used in this model.

4.1 Model Changes and Overview

The goal of this model is to determine the optimal timberlands planting and harvest rotation to sustain previous manufacturers as well as a new biorefinery. This work utilizes two novel ideas. The first is using a start up period before reaching the cyclical time rotation. This startup period allows the harvesters a buffer time to adapt their management strategies to accommodate a new biorefinery. The other uses two time indices to track land maturity. This also allowed us to maintain a linear model while still representing the complexity of biomass growth. The growth dynamic coefficients were calculated outside of the model over the range of ages in the biomass life cycle. See Tables 54 and 55 for these values.

This new model used a single level representation of the timberlands supply chain by combining the objectives and constraints of the bilevel model from Chapters 2 and 3. To implement the new scheduling aspect of the problem, harvester sizes were set by acreage instead of by biomass availability amounts. The harvest level index from the previous models was used to discourage over harvesting of materials. We felt that the dynamic aspect of the problem made it unnecessary since saving material for future time periods will be considered. The behavior of the lumber mills and biorefineries remained mostly the same. To create a more complete representation of the supply chain, a pulp mill was added to the model. The pulp mill utilizes fiber logs to produce pulp.

To accommodate for biofuel production, any changes in planting and harvesting strategies must be made years before the facility begins production. This is due to the long time requirement for timber biomass to mature. Therefore, in our model, we implemented a 10 year start-up period to adapt the cyclical planting rotation. While the biorefinery size and location decisions remained the same, investment timing was a new decision made in the start up period. With biofuel production as new production, switchgrass was also made an available planting alternative. Also, the age of the harvest zones determine the types of biomass available. In addition to switchgrass, three more biomass types were added to the model: thinnings, prunings, and fiber logs.

4.2 Mathematical Representation

This section discusses the mathematical representation of the developed cyclic model. Definitions of the indices, variables, and parameters are shown in Tables 46 to 51. Some of these definitions have changed from Chapters 2 and 3. For example, index h now describes the type of land planted.

4.2.1 Basic Constraints

Several restrictions on variables must be noted before discussion of the state equations and constraints. These restrictions prevent unrealistic behavior in the system. Within these models, it should be noted that two time indices are used: present time (t) and origin time (τ). Origin time tracks the year at which land is planted on. Therefore, land maturity can be determined with $t - \tau$.

$$x_{iht\tau} = 0 \ \forall \ i, h \text{ for } t - \tau \notin [\chi_{min} \ \chi_{max}] \tag{46}$$

$$x_{iht\tau} \geq 0 \ \forall \ i, h \text{ for } t - \tau \in [\chi_{min} \ \chi_{max}] \tag{47}$$

$$\hat{x}_{iht\tau} = 0 \ \forall \ i \text{ for } h = \text{timber}, t - \tau \notin [8 \ 12] \tag{48}$$

$$\hat{x}_{iht\tau} \geq 0 \ \forall \ i \text{ for } h = \text{timber}, t - \tau \in [8 \ 12] \tag{49}$$

Table 46: Index Information [Chapter 4]

Description	Index	Size	Range
Harvesting Zone	i	21	1 - 21
Manufacturer	m	9	Lumber Mills [1 8] Pulp Mill [9]
Biomass Type	k	7	Saw Logs Fiber Logs Thinnings Prunings Logging Residuals Shavings Grass
Land Type	h	3	Timber Switchgrass Thinned Land
Product Type	l	3	Lumber Boards Pulp Gasoline
Biorefinery Capacity	u	3	Small Medium Large
Biorefinery Location	e	3	Loc. 1 - Loc. 3
Present Time	t	31	[0 30]
Origin Time	τ	51	[-20 30]

Table 47: State Variables

$S_{iht\tau}$	Occupied land in zone i of type h planted at time τ at present time t
U_{it}	Unplanted land in zone i at present time t

Table 48: Decision Variables [Chapter 4]

P_{iht}	quantity to plant land type h in zone i at time t
$x_{iht\tau}$	full harvest amount of zone i of land type h planted at time τ at present time t
$\hat{x}_{iht\tau}$	thinning amount of zone i of land type h planted at time τ at present time t
y_{imklt}	purchase quantity from harvester i by manufacturer m of biomass type k for product l at present time t
$\hat{y}_{m'mklt}$	purchase quantity from manufacturer m' by manufacturer m of biomass type k for product l at present time t
π_{ikluet}	purchase quantity from harvester i of biomass type k for product l at location index e and capacity index u at present time t
$\hat{\pi}_{mkluet}$	purchase quantity from manufacturer m of biomass type k for product l at location index e and capacity index u at present time t

Table 49: Binary Decision Variables [Chapter 4]

z_{iht}	binary variable that controls the harvester i's activity for harvesting land type h at present time t
$\hat{z}_{iht}$	binary variable that controls the harvester i's activity for thinning land type h at present time t
$\theta_{ih\tau}$	binary variable that controls the harvester i's activity for planting land type h at origin time τ
D_{uet}	binary decision of biorefinery capacity index u of at location e at present time t

Table 50: Parameters [Chapter 4]

α_{kl}	conversion factor for biomass type k into product l
G_m	maximum capacity of manufacturer m
A_{ml}	capacity of product l that must be produced from manufacturer m
$\gamma_{kk'}$	fraction of resources of biomass type k' obtained from manufacturing biomass type k
κ	number of biorefineries that can be introduced to the system
B_i	land area of zone i
ω	tolerance for cyclical constraint for beginning and end states
H	minimum fraction of harvested land
β_{ue}	capital cost of biorefinery of capacity u at location e
$\hat{G}_u$	maximum capacity of biorefinery for capacity index u
R_i	minimum fraction of planting area of harvester i

Table 51: Time Dependent Parameters

c_{ikt}	selling price from harvester i for biomass type k at present time t
$\hat{c}_{mkt}$	selling price from manufacturer m for biomass type k at present time t
p_{mlt}	selling price from manufacturer m of product l at time t
δ_t	discount factor at present time t
o_{ikt}	selling price of harvester i's biomass type k to external buyers at present time t
$\hat{p}_{luet}$	selling price of biorefinery at location e of capacity u of product l at present time t
$\rho_{kht-\tau}$	Density from harvesting material type k from land type h at age $t-\tau$
$\hat{\rho}_{kht-\tau}$	Density from thinning material type k from land type h at age $t-\tau$
f_{iht}	cost per acre of harvesting land type h at harvester i
$\hat{f}_{iht}$	cost per acre of thinning land type h at harvester i
$\bar{c}_{iht}$	price of planting in zone i of land type h
s_{imt}	shipping cost from harvester i to manufacturer m
$\bar{s}_{met}$	shipping cost from manufacturer m to biorefinery at location e
$\breve{s}_{m'mt}$	shipping cost from manufacturer m from manufacturer m'
n_{mt}	processing cost per unit of material for manufacturer m
$\hat{n}_{ut}$	processing cost per unit of material for biorefinery of capacity index u
$\hat{s}_{iet}$	shipping cost from harvester i to biorefinery at location index e

Table 52: Harvest Age $(t-\tau)$ to Harvest Land Types

Land Type (h)	Minimum Harvest Age (χ_{min})	Maximum Harvest Age (χ_{max})
Timber	8	12
	18	22
Switchgrass	2	10
Thinned Land	18	22

This model used a more complex representation for timberlands management behaviors than models typically developed for this application. Land types and age restrictions for harvesting are shown in Table 52. Both $x_{iht\tau}$ and $\hat{x}_{iht\tau}$ can be non-zero within these ranges (Equations 46 - 49). The decision to plant timber (h = timber) yields two time periods of harvest. From 8 to 12 years, young timber can be fully harvested or thinned (changing the land to h = thinned) to provide more space for the maturing trees. After about 20 years, the trees are large enough to harvest. Furthermore, thinning the timberlands is a decision for the harvesters. Timberlands can remain unthinned; however, due to crowding, the trees cannot reach the size that those in thinned land can, providing a different distribution of biomass types. Unthinned timberlands can provide larger amounts of biomass but may not be able to provide as much specific material that production facilities require.

The decision to plant switchgrass with timber leads to a different behavior. For this planting arrangement, every other row of trees is instead a row of switchgrass. After 2 years, harvestable grass type biomass is provided yearly until the 10 year point. Afterwards, the tree canopy becomes too large, blocking out sunlight and hindering future growth. The remaining layout of trees was the same as a post-thinned timberlands.

Further infeasible actions must be prevented by restricting decision and state variables. To prevent a negative and 0 age, $S_{iht\tau}$, $x_{iht\tau}$, $\hat{x}_{iht\tau}$ are 0 when $t \leq \tau$. To prevent incorrect thinning decisions, $\hat{x}_{iht\tau} = 0$ when h was switchgrass or thinned land. Thinned land could only be generated from other land types, so $P_{iht} = 0$ when h = thinned land. Finally, planting decision P_{iht} can only be non-zero when $t = \tau$ for the constraint in which it's present. Since index τ is the origin time of planting, it makes sense that the planting decision can only occur when the indices are equal.

Also, all variables except for $x_{iht\tau}$, z_{iht}, $\hat{x}_{iht\tau}$, $\hat{z}_{iht}$ $S_{iht\tau}$, U_{it}, and D_{uet} are 0 at the final time point. This is due to the setup of the time schedule. The problem ends at the beginning of the final year. Therefore, state variables can have a value, since they describe the beginning of the state. Harvest and thinning variables can contain a value since the decisions with index $t+1$ are made at time t. Conversely, this means variables $x_{iht\tau}$ and $\hat{x}_{iht\tau}$ are 0 in the first year.

These restrictions will apply to the state equations, constraints, and objective function discussed in the next section.

4.2.2 State Equations

In this model, the state variables (U_{it} and $S_{iht\tau}$) describe the state of the timberlands. U_{it} is the amount of unplanted land for harvest zone i at time t. $S_{iht\tau}$ is the amount of land of type h at harvest zone i at present time t planted at origin time τ. The amount of unplanted land is calculated by Equation 50, where B_i is the total land capacity of harvest zone i. Also, all state variables are constrained to be non-negative (Equation 51).

$$U_{it} = B_i - (\sum_h P_{iht} + \sum_\tau S_{iht\tau}) \; \forall \; i, t \tag{50}$$

$$S_{iht\tau} \geq 0 \; \forall \; i, h, \text{ for } t \geq \tau \tag{51}$$

The different behaviors of switchgrass and timber require separate state equations. Also, a distinction is made for thinned land due to different growth dynamics of thinned versus unthinned lands.

4.2.2.1 Timber State Equations

Equation 52 is the state equation governing land type $h =$ timber. The value of the next state is determined by the amount of land harvested $x_{iht+1\tau}$ and thinned $\hat{x}_{iht+1\tau}$ subtracted from the value of the present state. The harvest decisions can only occur at the maturities shown in Table 52.

$$S_{iht+1\tau} = S_{iht\tau} - x_{iht+1\tau} - \hat{x}_{iht+1\tau} \; \forall \; i, t, \tau \text{ for } h = \text{timber} \tag{52}$$

At intervals when $t = \tau$, Equation 52 becomes Equation 53, adding planting decision P_{iht}.

$$S_{iht+1\tau} = S_{iht\tau} - x_{iht+1\tau} - \hat{x}_{iht+1\tau} + P_{iht} \; \forall \; i \text{ for } h = \text{timber}, t = \tau \tag{53}$$

4.2.2.2 Switchgrass State Equations

State behavior for switchgrass type land is shown in Equations 54 through 56. Equation 54 defines the amount of land planted. Equation 55 represents the behavior of the first 9 years of planted switchgrass. A harvest term $x_{iht\tau}$ does not exist in the state equation. Since a consistent amount of material is generated each year of its lifespan, harvesting does not subtract from the state variable. Equation 56 removes the switchgrass type land after 10 years. This land is converted to thinned land, which is seen in Equation 58.

$$S_{iht+1\tau} = P_{iht} \; \forall \; i \text{ for } h = \text{switchgrass}, t - \tau = 0 \tag{54}$$

$$S_{iht+1\tau} = S_{iht\tau} \; \forall \; i \text{ for } h = \text{switchgrass}, 0 < t - \tau < 10 \tag{55}$$

$$S_{iht+1\tau} = 0 \; \forall \; i \text{ for } h = \text{switchgrass}, t - \tau \geq 10 \tag{56}$$

Equation 57 must also be included to constrain switchgrass harvesting. Due to the nature of switchgrass, biomass is provided on a yearly basis over a duration. Harvesting has no impact on the amount it provides, so the state variable is not changed by the harvesting decision. Equation 57 sets an upper bound that ensures that the amount of switchgrass harvested does not exceed its planted land.

$$x_{iht+1\tau} \leq S_{iht\tau} \; \forall \; i, \text{ for } h = \text{ switchgrass}, \; t \geq \tau \tag{57}$$

4.2.2.3 Thinned Land State Equations

Equation 58 is the state equation for thinned land. Thinned land can only be generated from thinning the timber land type or choosing the switchgrass planting decision. After 18 years, the thinned land can be fully harvested.

$$\begin{aligned} S_{iht+1\tau} =& S_{iht\tau} - x_{iht+1\tau} + \hat{x}_{ih't+1\tau} \; \forall \; i, t, \tau \\ & \text{for } h = \text{thinned}, \; h' = \text{timber} \end{aligned} \tag{58}$$

When the switchgrass land has matured 10 years ($t - \tau = 10$), Equation 58 becomes Equation 59, which adds the switchgrass state term $S_{ih"t\tau}$.

$$\begin{aligned} S_{iht+1\tau} =& S_{iht\tau} - x_{iht+1\tau} + \hat{x}_{ih't+1\tau} + S_{ih"t\tau} \; \forall \; i \\ & \text{for } h = \text{thinned}, \; h' = \text{timber}, \; h" = \text{ switchgrass}, \; t - \tau = 10 \end{aligned} \tag{59}$$

4.2.3 Decision Variable Bounds

The following constraints cover the activity of certain decisions. Each of these constraints come in pairs, creating an upper and lower bound. These bounds prevent decisions with unrealistically small values. Equation 60 is the upper and lower bounds of the planting decision. Equations 61 and 62 control the harvesting and thinning decisions. In these equations, B_i is the total amount of land (acres) in zone i. R_i and H are ratios that are the minimum fraction of land that must be planted/harvested. R_i is specific to zone i. θ_{iht} controls the activity of the planting decision while z_{iht+1} and $\hat{z}_{iht+1}$ control the activity for the harvesting and thinning decisions.

$$R_i B_i \theta_{iht} \leq P_{iht} \leq B_i \theta_{iht} \; \forall \; i, h, t \tag{60}$$

$$HB_i z_{iht+1} \leq \sum_{\tau} x_{iht+1\tau} \leq B_i z_{iht+1} \; \forall \; i, h, t \tag{61}$$

$$HB_i \hat{z}_{iht+1} \leq \sum_{\tau} \hat{x}_{iht+1\tau} \leq B_i \hat{z}_{iht+1} \; \forall \; i, h, t \tag{62}$$

4.2.4 Biorefinery Constraints

The biorefinery decision D_{uet} can only be made in the first 10 years. The number of biorefineries that can be built is limited to κ (Equation 63). Once the investment has been made, binary variable D_{uet} remains active (Equation 64). After the initial 10 years, the biorefinery decisions are fixed (Equation 65). Equation 66 ensures that only one biorefinery can exist at a location.

$$\sum_{u} \sum_{e} D_{uet} \leq \kappa \text{ for } 1 \leq t \leq 10 \tag{63}$$

$$D_{uet+1} \geq D_{uet} \; \forall \; u, e \text{ for } 1 \leq t < 10 \tag{64}$$

$$D_{uet+1} = D_{uet} \; \forall \; u, e \text{ for } 10 \leq t \leq 29 \tag{65}$$

$$\sum_{u} D_{uet} \; \leq 1 \; \forall \; e, \; \text{ for } 1 \leq t \leq 10 \tag{66}$$

Equation 67 determines the manufacturer biomass byproduct availability. This is determined by the amount of material processed by manufacturers ($y_{imk'l't}$). The amount generated is determined by yield term $\gamma_{k'k}$. The pulp mill can also utilize the lumber mill byproduct. The term $\hat{y}_{mm'klt}$ represents the amount of material that the pulp mill (m') purchases from manufacturers (m).

$$\sum_{l} \left(\sum_{u} \sum_{e} \hat{\pi}_{mkluet} + \sum_{m'} \hat{y}_{mm'klt} \right) \leq \sum_{k'} \gamma_{k'k} \sum_{i} \sum_{l'} y_{imk'l't} \; \forall \; m, k, t \tag{67}$$

Equation 68 is the capacity constraint for the biorefineries. The capacity is controlled by biorefinery investment variable D_{uet}.

$$\sum_k \sum_l (\sum_i \pi_{ikluet} + \sum_m \hat{\pi}_{mkluet}) \leq D_{uet}\hat{G}_u \ \forall\ u, e, t \tag{68}$$

4.2.5 Manufacturer Constraints

The manufacturer constraints are given by Equations 69 - 71. Equation 69 is an upper bound limited by the facility capacity. Equation 70 is the demand from retailers on the manufacturer. Equation 71 is the biomass availability constraint. It restricts the amount of biomass that can be purchased by both manufacturers and biorefineries by the amount of material harvested.

$$\sum_k \sum_l (\sum_i y_{imklt} + \sum_{m'} \hat{y}_{m'mklt}) \leq G_m \forall\ m, t \tag{69}$$

$$\sum_k \alpha_{kl} (\sum_i y_{imklt} + \sum_{m'} \hat{y}_{m'mklt}) \geq A_{ml}\ \forall\ m, l, t \tag{70}$$

$$\sum_l (\sum_m y_{imklt} + \sum_u \sum_e \pi_{ikluet}) \leq \sum_h \sum_\tau (\rho_{kht-\tau} x_{iht+1\tau} + \hat{\rho}_{kht-\tau} \hat{x}_{iht+1\tau})\ \forall\ i, k, t \tag{71}$$

4.2.6 Cyclical Constraints

Equations 72 and 73 ensure that the cyclical conditions of the problem are met. T is the time period of the new cyclic schedule initial state, and its value is based off the duration of the transition period (if transition period is 10 years, the new cyclical initial state is given by the state variables at time $t = T = 10$. The final state variables occur at time $t = T + 20$ and can deviate from the initial state variables within tolerance ω. The τ indices for the final state must also also be adjusted by 20 years. These cyclical constraints are quite restrictive in that they require age and biomass type distribution among each harvest zone to be constrained within ω of the starting values.

$$S_{ih30\tau+20} \geq (1-\omega)S_{ih10\tau} \ \forall \ i, h, \tau \tag{72}$$

$$S_{ih30\tau+20} \leq (1+\omega)S_{ih10\tau} \ \forall \ i, h, \tau \tag{73}$$

4.2.7 Objective Function

The objective function for the model is the overall system profit. This means the distribution of value within the supply chain does not affect the overall value of the objective function. This is further discussed in Section 4.2.7.3. The revenues and costs described below are specific to each time period t. Each time period is adjusted for inflation and then discounted backwards by a factor of 5% to reflect current interest rates for the calculation of net present value. The inflation values were taken from [4].

4.2.7.1 Revenue For Each Time Period

Material harvested in excess to the demands of manufacturers and biorefineries within the supply chain system are sold externally at price o_{ikt} (Equation 74).

$$\begin{aligned} \text{External Revenue} = & \sum_i \sum_k o_{ikt} (\sum_\tau (\rho_{kht-\tau} x_{iht+1\tau} + \hat{\rho}_{kht-\tau} \hat{x}_{iht+1\tau}) \\ & - \sum_l (\sum_m y_{imklt} + \sum_u \sum_e \pi_{ikluet})) \end{aligned} \tag{74}$$

Both the biorefineries and manufacturers generate profits from selling their final products. Their revenues are calculated by Equations 75 and 76.

$$\text{Biorefinery Revenue} = \sum_l \sum_u \sum_e \hat{p}_{luet} \sum_k \alpha_{kl} (\sum_i \pi_{ikluet} + \sum_m \hat{\pi}_{mkluet}) \tag{75}$$

$$\text{Manufacturer Revenue} = \sum_m \sum_l p_{mlt} \sum_k \alpha_{kl} \sum_i y_{imklt} \tag{76}$$

4.2.7.2 Costs For Each Time Period

Costs within the system include harvesting costs (Equation 77), capital costs (Equation 78), planting costs (Equation 79), transportation costs (Equations 80-83), and processing costs (Equations 84 and 85). The capital costs in Equation 78 are a one time fee incurred if a biorefinery is built. Therefore, if variable D_{uet} is 1 when $t = T$ (the beginning of the new cyclic schedule), then the cost is included. Therefore, this part of the objective function is not reliant on t.

$$\text{Harvesting Costs} = \sum_i \sum_h \sum_\tau f_{iht} x_{iht\tau} + \hat{f}_{iht} \hat{x}_{iht\tau} \tag{77}$$

$$\text{Annualized Capital Cost} = \sum_u \sum_e \beta_{ue} D_{ueT} \tag{78}$$

$$\text{Planting Cost} = \sum_i \sum_h \bar{c}_{iht} P_{iht} \tag{79}$$

$$\text{Biorefinery Transportation Costs from Harvesters} = \sum_i \sum_e \hat{s}_{iet} \sum_k \sum_l \sum_u \pi_{ikluet} \tag{80}$$

$$\text{Manufacturer Transportation Costs from Harvesters} = \sum_i \sum_m s_{imt} \sum_k \sum_l y_{imklt} \tag{81}$$

$$\text{Manufacturer Transportation Costs from Manufacturers} = \sum_{m'} \sum_m \breve{s}_{m'mt} \sum_k \sum_l \hat{y}_{m'mklt} \tag{82}$$

$$\text{Biorefinery Transportation Costs from Manufacturers} = \sum_m \sum_e \bar{s}_{met} \sum_k \sum_l \sum_u \hat{\pi}_{mkluet} \tag{83}$$

$$\text{Manufacturer Processing Costs} = \sum_m n_{mt} \sum_i \sum_k \sum_l y_{imklt} \tag{84}$$

$$\text{Biorefinery Processing Costs} = \sum_u \hat{n}_{ut} \sum_e \sum_k \sum_l (\sum_i \pi_{ikluet} + \sum_m \hat{\pi}_{mkluet}) \tag{85}$$

4.2.7.3 Additional Objective Values

Equations 86 to 89 represent the costs of acquiring biomass materials between sectors. These costs are complemented by the revenue gained from sales by a different sector, which yields no effect on the value of the objective function. These equations are included for the calculations of profits specific to the harvesting, manufacturing, and biofuel sectors of the supply chain.

$$\text{Biomass Costs from Harvesters to Manufacturers} = \sum_i \sum_k c_{ikt} \sum_l (\sum_m y_{imklt}) \tag{86}$$

$$\text{Biomass Costs from Harvesters to Biorefineries} = \sum_i \sum_k c_{ikt} \sum_l (\sum_u \sum_e \pi_{ikluet}) \tag{87}$$

$$\text{Biomass Costs from Manufacturers to Manufacturers \& Biorefineries} = \sum_m \sum_k \hat{c}_{mkt} \sum_l (\sum_{m'} \hat{y}_{mm'klt} + \sum_u \sum_e \hat{\pi}_{mkluet}) \tag{88}$$

$$\text{Biomass Costs from Manufacturers to Biorefineries} = \sum_m \sum_k \sum_e \hat{c}_{mkt} \sum_l \sum_u \hat{\pi}_{mkluet} \tag{89}$$

4.3 Problem Generation and Case Studies

The two major decision makers in our system are the harvesters and manufacturers. In our case studies, the harvesters provide loblolly pine type biomass, which is assumed to have a life cycle of about 20 years. After 10 years, the timberlands can be maintained with row thinning, which removes every other row of trees [21]. This prevents congestion of the forestlands, allowing the remaining trees more space to grow. Timber from thinned land provide saw logs, fiber logs, and residuals. Unthinned timber also provide these types of biomass, but less saw logs are available due to limited growing space. Instead, a larger total amount of material is harvested, and larger percentage of fiber logs are provided. Material obtained from thinning decisions are thinnings, residuals and prunings.

The established manufacturers within the geographic scope of this problem are lumber mills and a pulp mill in the southeast of the United States. Lumber mills process saw logs into timber while pulp mills can use fiber logs, residuals, shavings, thinnings, and prunings to produce pulp. 90% is the minimum processing capacity for these established facilities. We felt that the established manufacturers should not have to sacrifice productivity for the biorefinery. These facilities could be contractually obligated to provide a consistent flow of product to retailers.

Introducing biofuel production to the supply chain could require alterations to the planting rotations, such as adding an alternative biomass crop of switchgrass. Switchgrass would provide an early consistent source of biomass for biofuel manufacturing. Also, instead of the more crowded timber planting strategy, every other row of trees could be replaced with rows of switchgrass. This means that row thinning is not needed because the switchgrass grows between the rows that are spaced further apart during the first 10 years before the canopy closes over it.

The model developed in these studies contained 21 harvest zones, 7 lumber mills, and 1 pulp mill. The system is limited to 1 biorefinery investment. The total land area is 724,000 acres. The case studies are assumed to begin at year 1974, and the historical inflation rates from [4] are used to accurately inflate the prices through the time horizon.

Two case studies were performed with this timberlands model. First, several different scenarios with varying initial states of the forest and land capacity were generated. These initial states were tested in the modified cyclic model to determine their viability to sustain biofuel production. For the second set of case studies, the duration of the transition period was varied to study if additional time would lead to different outcomes.

Initial states were generated by the model. Without biorefinery investment decisions, the 20 year cycle emulates established timberlands strategies. The problem was run without an initial state and biofuel production, and the initial state of the cycle was saved as an initial state for the case studies.

4.4 Results

Optimization was performed in GAMS using the CPLEX solver on the Georgia Institute of Technology Joe cluster. Information about the specifications of the cluster can be found at [1].

4.4.1 Initial State Generation

Figure 11 shows a generated established cycle for our timberlands system and Figure 10 is the legend for all of the following figures. This cycle was generated by optimizing a 20 year cyclic model without biorefinery decisions. The established schedule generated in this section and Section 4.4.2 would be used in the system transition problems.

The 3 smaller plots on the left of Figure 11 are the land amounts utilized for timber and thinned lands. The amount of unplanted land is also shown. The x-axis range of the graphs go from year 0 to 21 to show the value of the states at year 20;

Figure 10: Legend

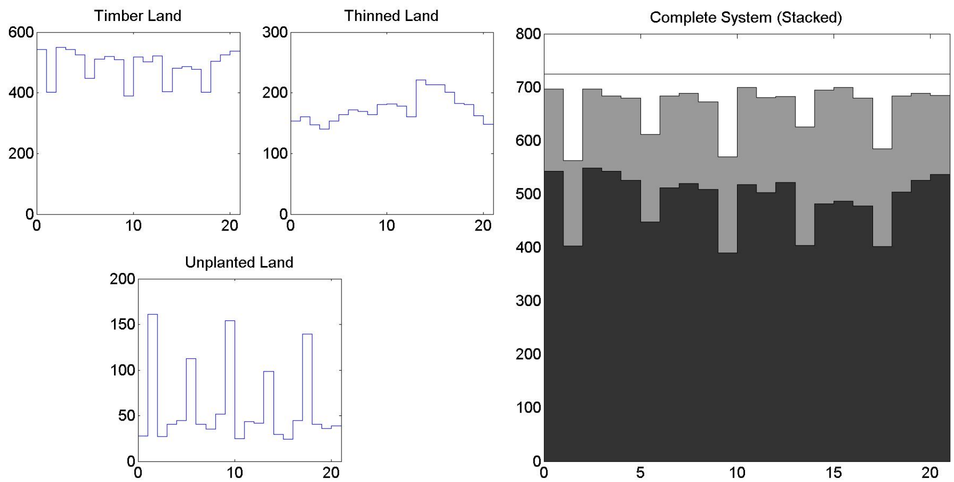

Figure 11: Established Timberlands Schedule (unconstrained)

the values of year 20 are shown in the bar between years 20 and 21. The larger figure on the right is a stacked representation displaying the distribution of land at each time point.

Table 53: Figure Axes

Axis	Description	Symbolic Description	Units
x	Time Period	t	year
y	Land Usage	$\sum_i \sum_\tau S_{iht\tau}$	10^3 acres

This initial solution shows inconsistent behavior, particularly at years 1, 5, 9, 13, and 17. These dips occurred through decisions to harvest large amounts of material in the previous year, leaving the next state at a lower land usage. When viewing the subplots, it can be seen that the large dips occurred only in the timber type land while the thinned land stays relatively consistent. What happened in these scenarios was that the harvested thinned land was replenished by thinning decisions. Another interesting pattern was that these dips occurred at intervals of 4 years, but this may be just due to the specifics of our case study.

Timberlands management may want to avoid these large dips in land usage, so smoothing constraints were introduced to promote consistent harvesting and thinning at each time period (Equation 90 & 91). We also felt that these constraints were reasonable in that they enforced consistent behavior through the time horizon. The upper bound values of these constraints were determined by averaging the amount of material harvested in each time period in an unconstrained problem.

$$\sum_i \sum_h \sum_\tau x_{iht\tau} \leq X \ \forall \ t \tag{90}$$

$$\sum_i \sum_h \sum_\tau \hat{x}_{iht\tau} \leq \hat{X} \ \forall \ t \tag{91}$$

$$X = \hat{X} = 72 \text{ acres} \tag{92}$$

3 cases were run around these constraints: “Fully constrained” case where both smoothing constraints of Equations 90-91 are active, “harvester constrained” case where only the harvesting constraint (Equation 90) is active, and “unconstrained” case with neither constraint active.

For the constrained problems, the stacked representations show a much more consistent behavior for land usage; the severe drops in planted land do not exist in these systems. While no clear difference in consistency can be seen between the harvester constrained and fully constrained cases, on average, the fully constrained case utilizes more land through the 20 year period. These 3 case studies were solved within 10% optimality.

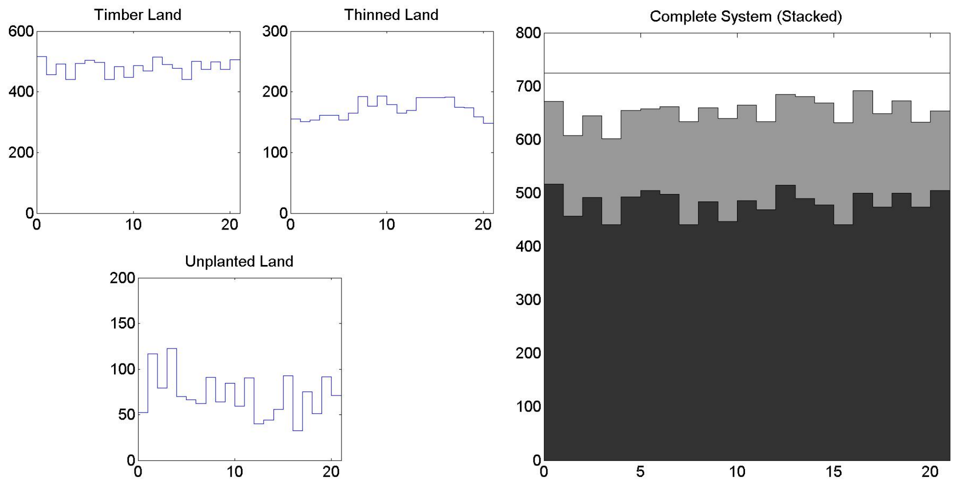

Figure 12: Established Timberlands Schedule (harvester constrained)

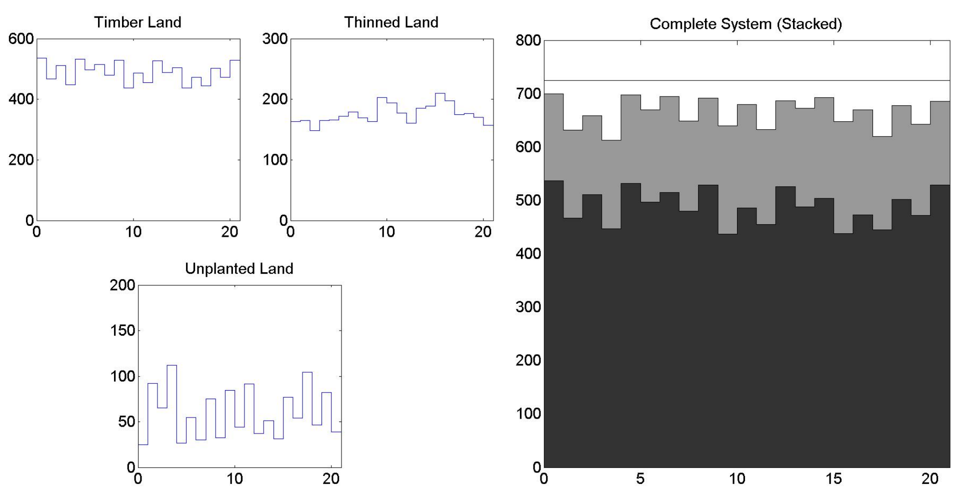

Figure 13: Established Timberlands Schedule (fully constrained)

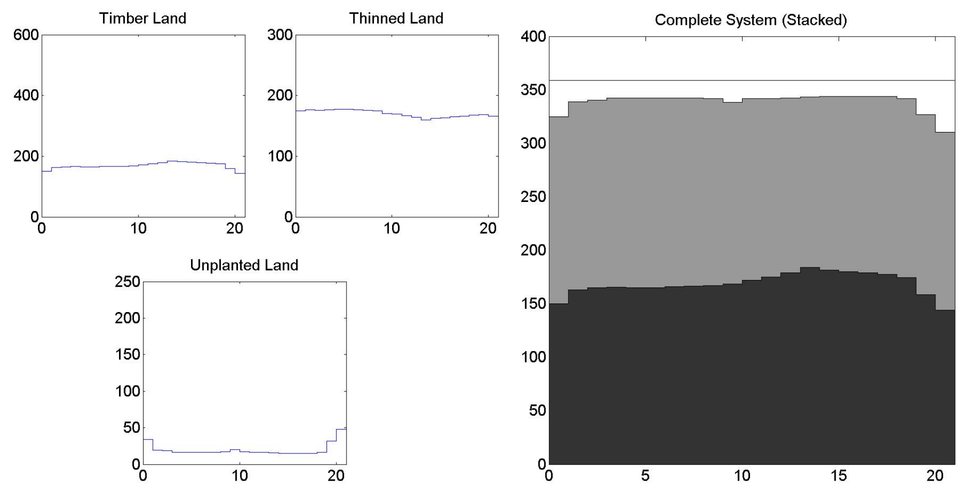

Figure 14: Schedule for Variable Land Model

4.4.1.1 Variable Land Study

A study was performed that minimized land usage but still fulfilled the demands of the system throughout the 20 year cycle. This was done by introducing a new set of binary variables that controlled the activity of the harvesters. In the objective function, these variables were heavily penalized for taking on a value of 1. This would encourage the solver to deactivate as many harvesters as possible.

For the first study, the problem was unconstrained. Not only were the smoothing constraints not present, but the planting, harvesting, and thinning upper and lower bounds were also removed (Equations 60 - 62). Solving this problem revealed that 5 harvesters was the absolute minimum number required to keep the manufacturers running at 90% capacity. These 5 harvesters had a total of 0.359 million acres of land, whereas the total system had 0.724 million acres.

Figure 14 shows the results of the unconstrained variable land study. The land usage remained consistent with very small increases and decreases except at the beginning and end. The results from this study was used as a lower bound for the constrained problem.

This set of 5 harvesters was used in 2 ways to determine the initial state for a

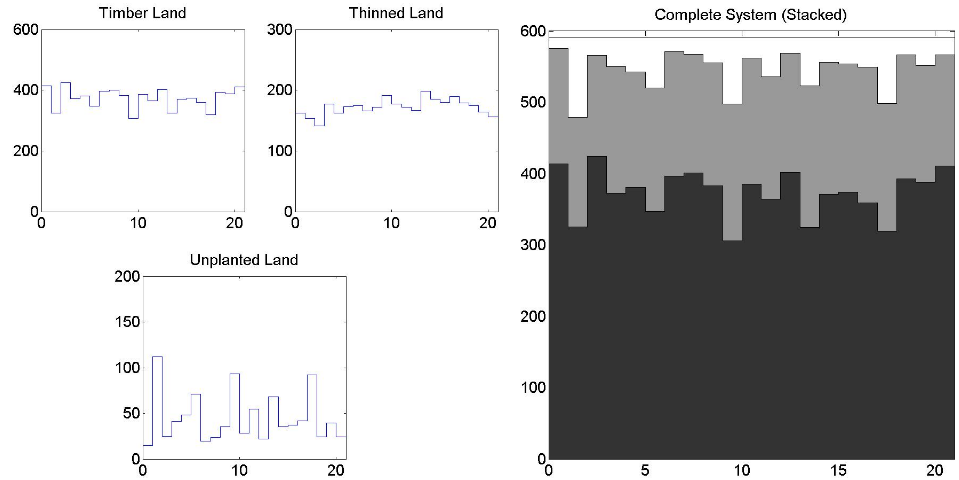

Figure 15: Established Timberlands Schedule (moderate land, unconstrained)

system with low timber resources. For the first method, the variable land model was re-used and the constraints were reintroduced. For the second method, the initial set of case studies in Section 4.4.1 was run with this set of 5 harvesters. After 8 hour calculation times for both methods, none of the problems converged to within 40% of optimality. The reintroduction of Equations 60 - 62 severely tightened the problem and created a smaller feasible region that was making difficult for GAMS to find a solution.

4.4.2 Moderate Land Capacity Initial State Generation

Another set of initial conditions was generated by setting up a system with moderate levels of land. This system contained the 13 largest harvesters, including the 5 from the variable land study. This system was run in the 3 smoothing constraint cases and solved to within 15% optimality. Less land capacity yielded a more difficult problem to solve, which was why the solution tolerance was higher. Figures 15 through 17 shows the initial state problems under moderate land availability.

The unconstrained case was similar to scenario with normal land capacity, except the drops in year 5 and 13 are not as severe. It seems that the major decreases in land

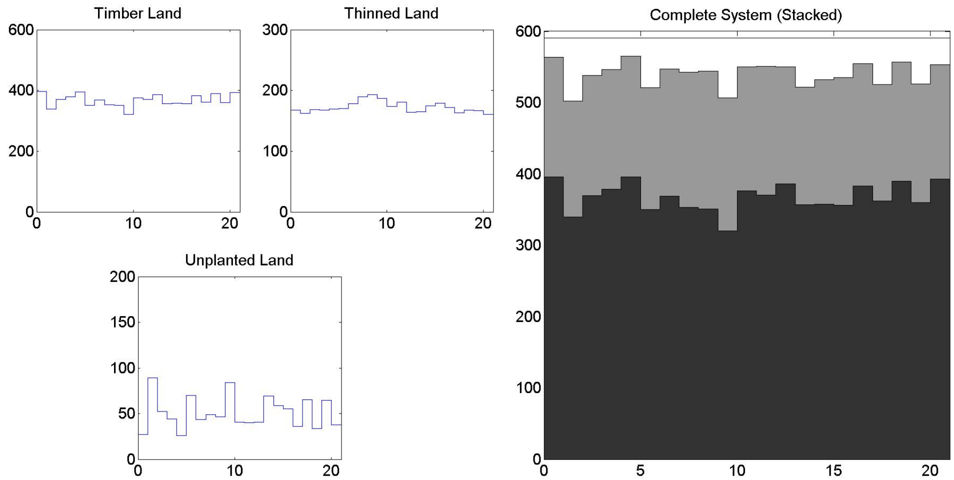

Figure 16: Established Timberlands Schedule (moderate land, harvester constrained)

usage occur at intervals of 8 years with smaller decreases in between. The subplots for thinned and timber land usage are much more consistent than their normal land scenario counterparts. With less land available in the system, the managers may require more consistent land management to provide to the manufacturers.

The constrained problems were more consistent than the unconstrained. The harvester constrained problem seemed to have more periods of level land usage or gradual increases of land usage (years 6-9, 10-13, 13-16) while the fully constrained problem seemed to show larger increases and decreases every year.

The stacked representations in the previous figures do not give a complete view of the initial state, particularly the age distribution of the land at that time. Figure 18 shows the age distribution of the lands at the initial state for normal land availability. Thinned land was composed of ages 11-20 years. Timber land had ages of 1-20 years with the majority of land 12 years and younger. Large amounts of 19 year old thinned land and 11 year old timber land are seen for the unsmoothed case. The maturity of these lands are within the range of harvesting and thinning, which explains the large drop in land use in year 1. The peaks of the constrained cases never reached the level of the unconstrained case. The restriction from the constraints could limit the height

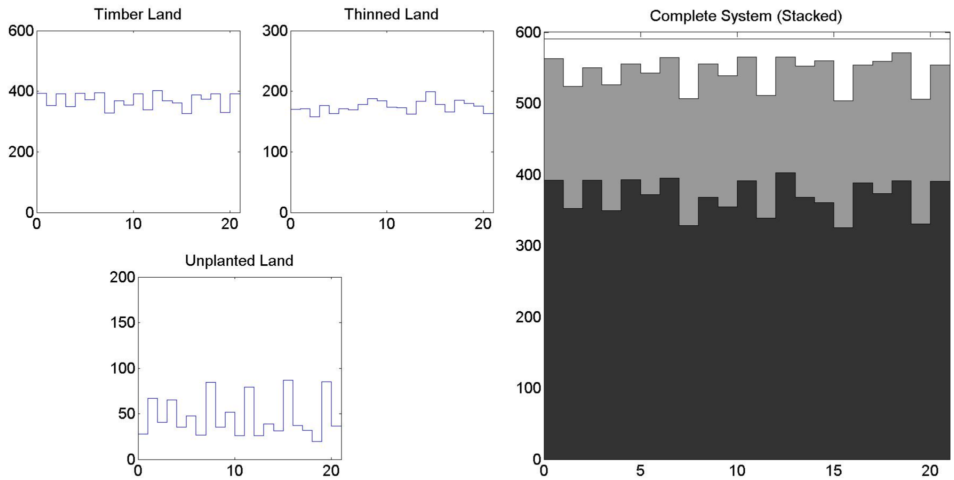

Figure 17: Established Timberlands Schedule (moderate land, fully constrained)

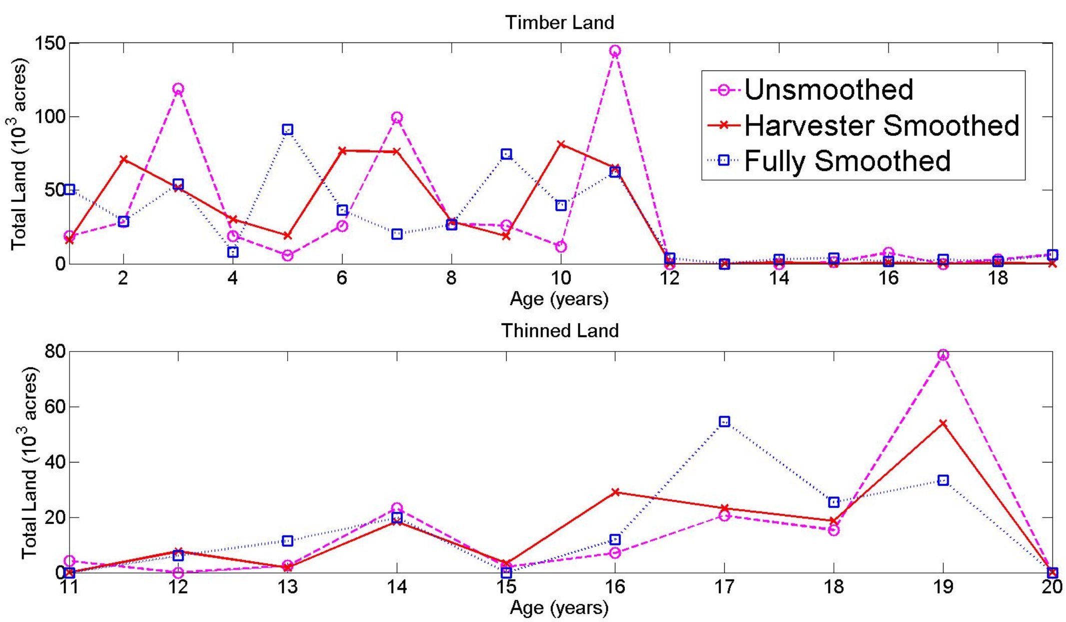

Figure 18: Initial State Age Distribution (normal land)

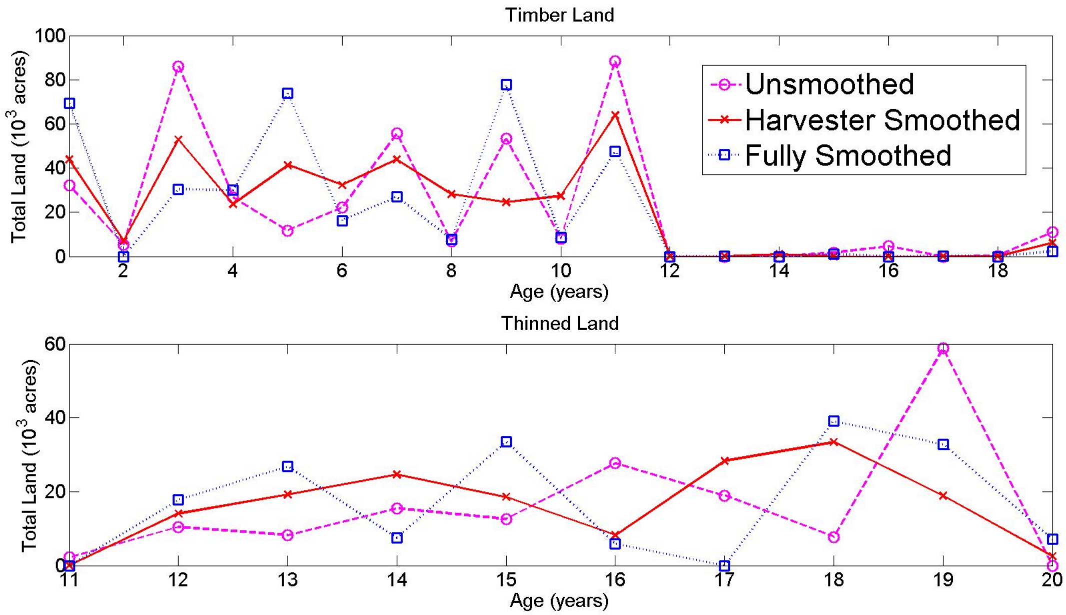

Figure 19: Initial State Age Distribution (moderate land)

of the these peaks.

Figure 19 shows the initial state age distribution of the moderate land cases. Again, for the unconstrained case, large amounts of land can be seen at years 11 (timber) and 19 (thinned). When comparing the age distributions of the thinned and timber type lands, the timberland distribution was much more erratic compared to the thinned land solution for both the normal and moderate land cases. The peaks in the timber land graphs corresponded to drops in land use in the previous figures.

4.4.3 Schedule Adaptation Results

The 6 sets of initial conditions generated above (6 different combinations of smoothing constraints and land availability) were used in the schedule adaptation studies. In these studies, a 10 year transition period is used to adapt the harvest schedule to incorporate a new biorefinery. The land availability of the system is the same as the situation from which the initial state was generated, but the smoothing constraints were not included. Because the system was changing to a new harvest schedule, the

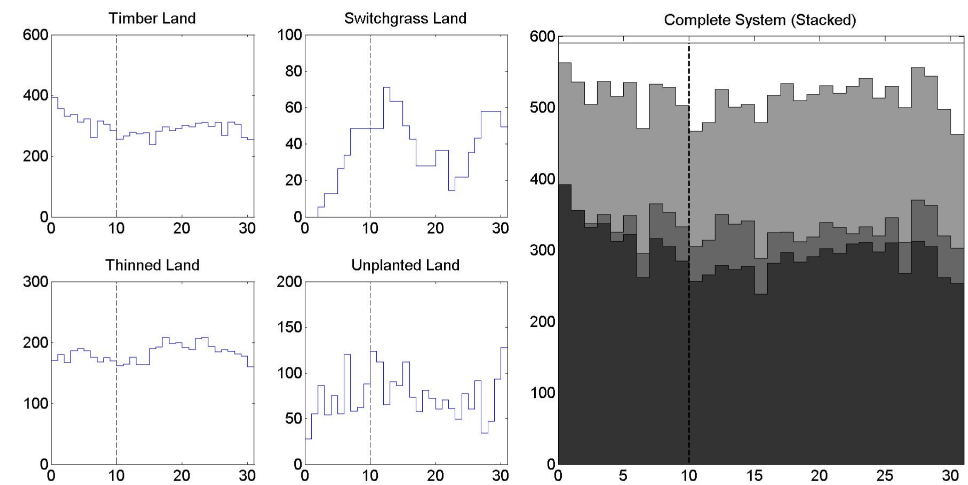

Figure 20: 10 Year Transition Timberlands Schedule (moderate land, fully constrained initial conditions)

previous bounds could be invalid. Therefore, initial unsmoothed solutions could be used to set new bounds for the constraints.

For these 6 sets of initial conditions, 5 converged on a solution. The normal land cases converged within 10% optimality. The moderate land cases converged within 30% except for the fully constrained problem with an 8 hour calculation time. To obtain a solution, this model was split into two cases: one with biorefinery investment enforced and one with no biorefinery. The solutions were compared, and the biorefinery enforced model provided a higher best possible objective value in GAMS. This solution, as well as the harvester constrained normal land and unconstrained moderate land cases yielded an optimal solution with a biorefinery. These cases are shown in Figures 20 to 22.

For these cases, timberland usage decreased during the transition period. The thinned land usage at the new cyclic starting year was consistent with the initial state. After the clearing of timber type land, it was transitioned towards switchgrass. Figure 20 and 22 start the new cycle with less switchgrass land than Figure 21. We believe this can be attributed to the starting behavior of the 30 year period. Figure

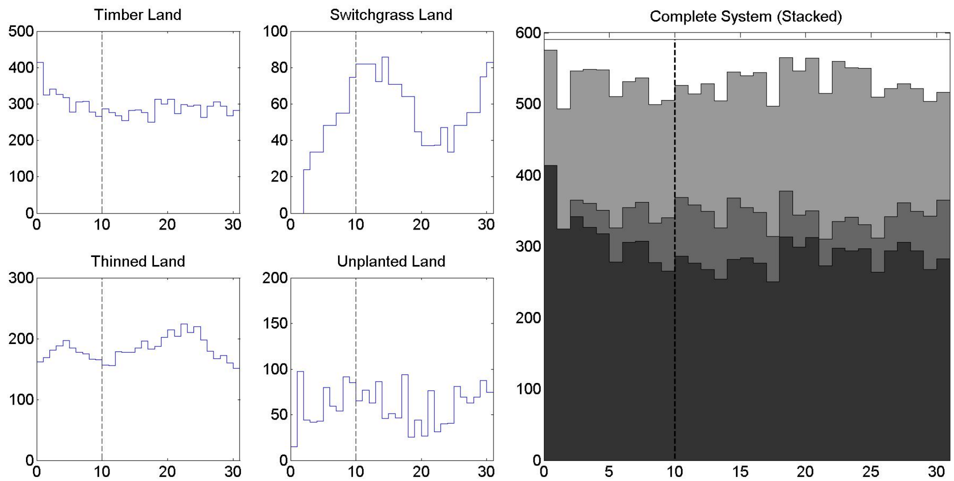

Figure 21: 10 Year Transition Timberlands Schedule (moderate land, unconstrained initial conditions)

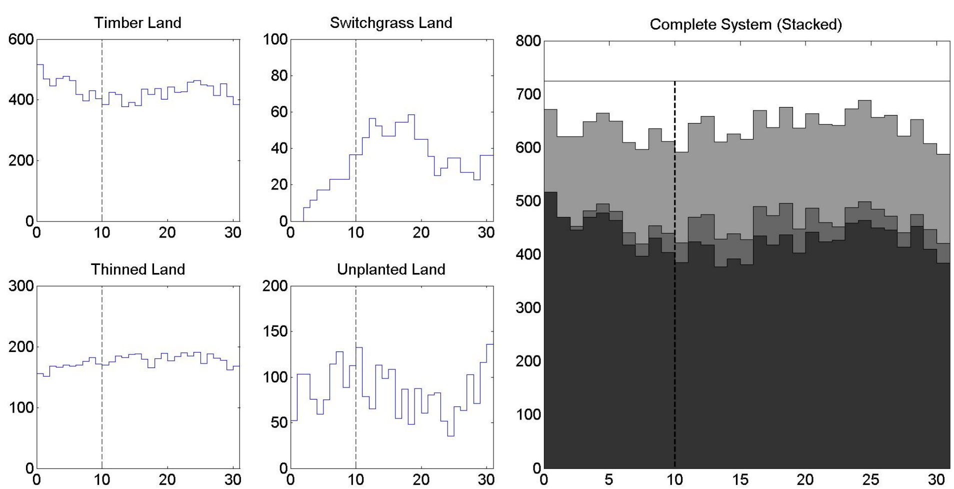

Figure 22: 10 Year Transition Timberlands Schedule (normal land, harvester constrained initial conditions)

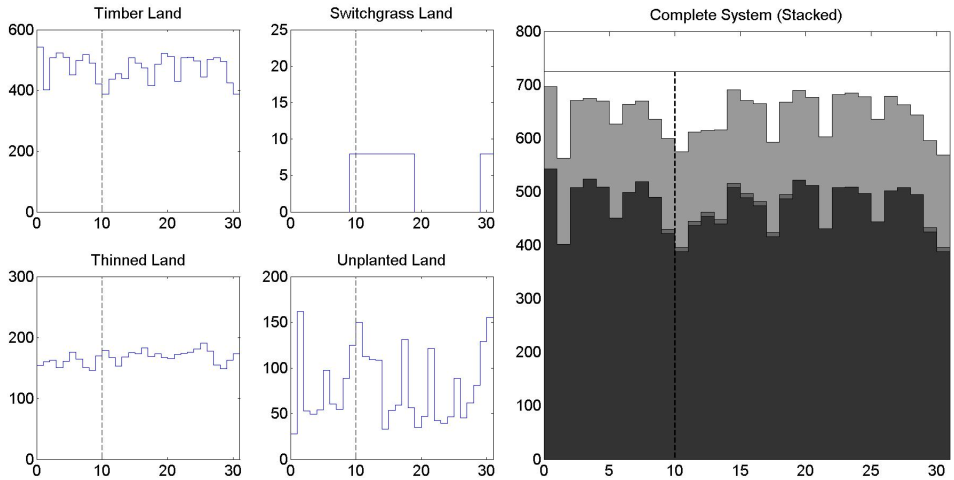

Figure 23: 10 Year Transition Timberlands Schedule (normal land, unconstrained initial conditions)

21 starts with a large drop in land usage, thereby freeing more space for a larger initial planting decision for switchgrass. In all three cases, the switchgrass usage dips around the middle of the new cycle before rising again to satisfy the cyclic constraint.

For the 3 cases where a biorefinery investment was not optimal, a second case was studied where a biorefinery investment was enforced. Figures 23 and 24 are the two sets of results from the unconstrained normal land scenario, Figures 25 and 26 are the results from the fully constrained normal land case, and Figures 27 and 28 are the results from the harvester constrained moderate land case. For the biorefinery enforced cases, the system also transitioned land to switchgrass. The unconstrained case had a larger initial drop, which was transitioned to larger amounts of switchgrass land.

Upon examination of the land type subplots, it can be seen that, for all presented cases, both normal and moderate land systems, the initial state of thinned land is at a consistent level, around 150,000 to 180,000 acres. Furthermore, it seems that this initial state is a relative lower bound for thinned land usage through the time period. Harvesters may need to maintain this level of thinned land to provide consistent

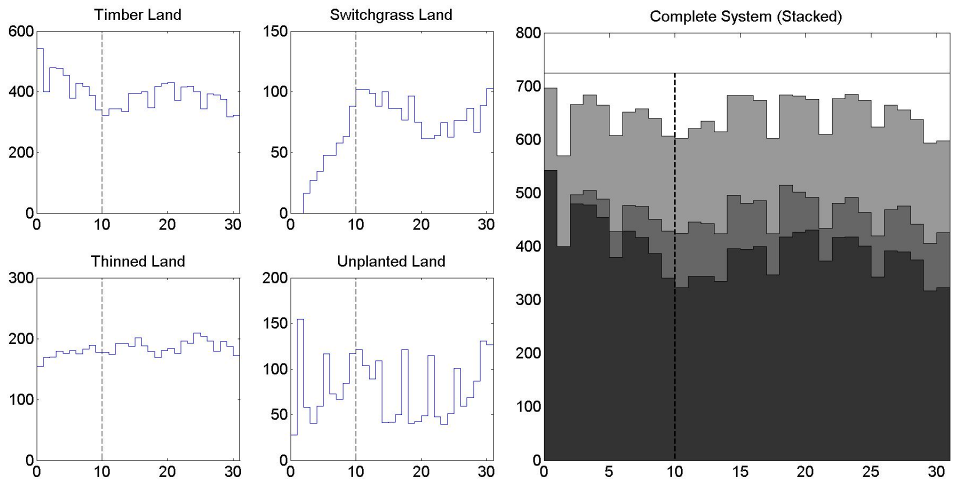

Figure 24: 10 Year Transition Timberlands Schedule (normal land, unconstrained initial conditions, enforced biorefinery)

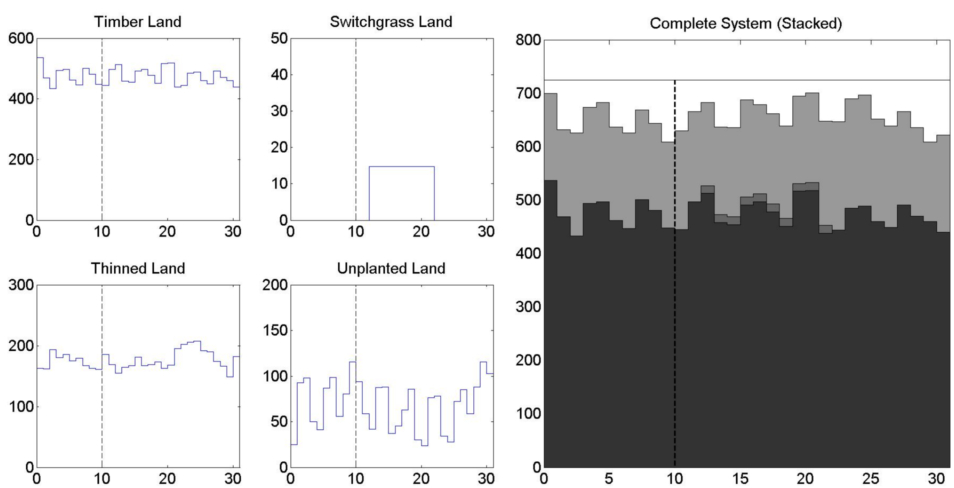

Figure 25: 10 Year Transition Timberlands Schedule (normal land, fully constrained initial conditions)

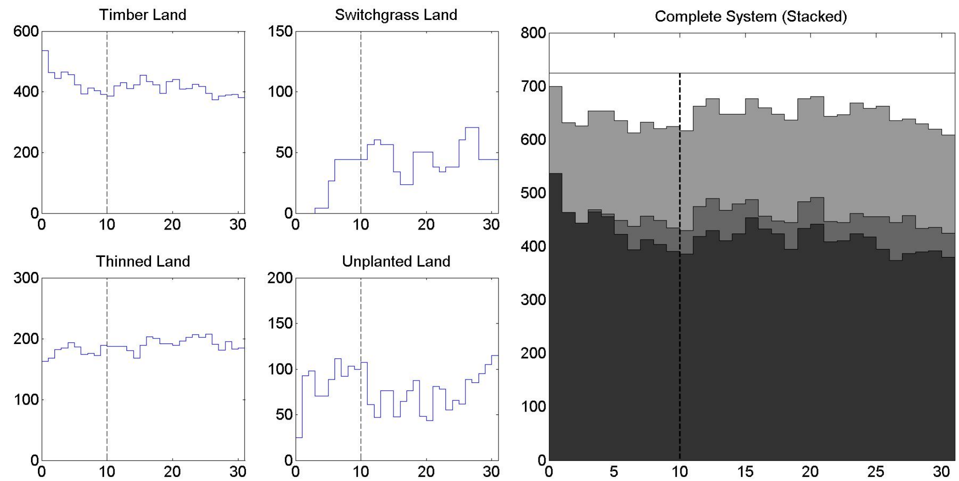

Figure 26: 10 Year Transition Timberlands Schedule (normal land, fully constrained initial conditions, biorefinery enforced)

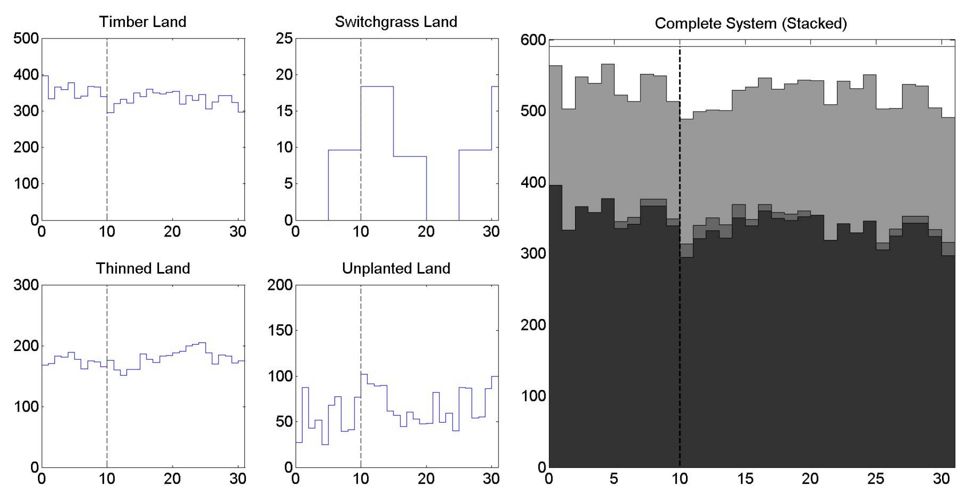

Figure 27: 10 Year Transition Timberlands Schedule (moderate land, harvester constrained initial conditions)

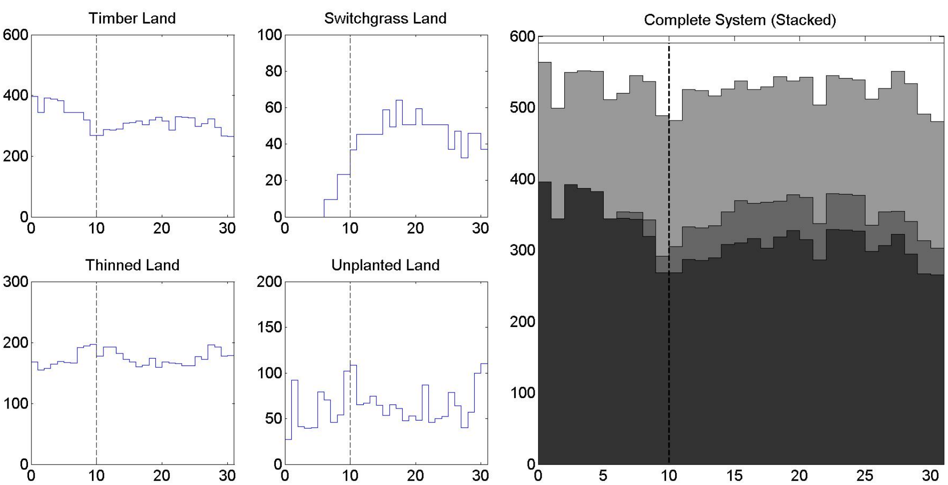

Figure 28: 10 Year Transition Timberlands Schedule (moderate land, harvester constrained initial conditions, biorefinery enforced)

biomass flow to the manufacturers.

When viewing Figures 20 to 28 alongside Figures 18 and 19, it is not clear why certain scenarios chose biofuel investments. There is no apparent correlation between the initial state and the choice of biofuel investment. This implies that there is a more complex relationship between initial state and biorefinery decision. Further study is needed to better understand this interaction.

4.4.4 Transition Time Studies

An additional study was conducted where the transition time period length was varied. The initial 10 years was extended to 15 and 20 year intervals. All of the previous initial states were used, yielding 12 different cases. For the 15 year calculation times, only 4 solutions were obtained through GAMS. The other 2 did converge in reasonable time. These results are shown in Figures 29 through 32.

Figures 29 through 31 show a 15 year transition period scenario for initial states that did not invest in biofuels with a 10 year transition period. With 5 additional years, all 3 of these cases yielded an investment in a biorefinery. Similar to previous results, a large portion of timber type land was converted to switchgrass type land.

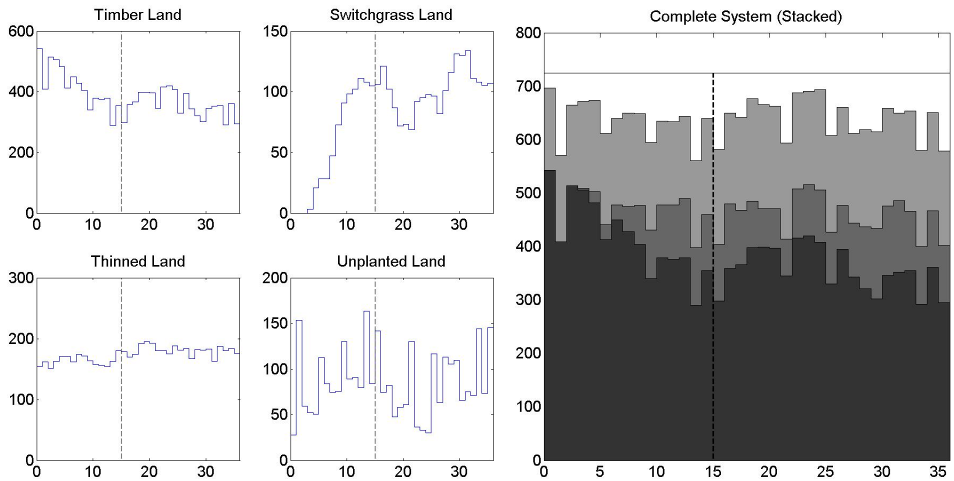

Figure 29: 15 Year Transition Timberlands Schedule (normal land, unconstrained initial state)

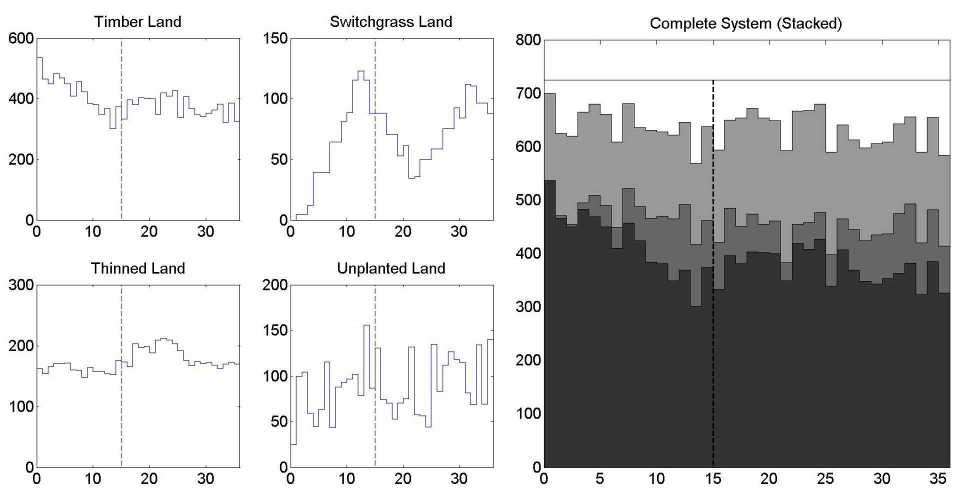

Figure 30: 15 Year Transition Timberlands Schedule (normal land, fully constrained initial state)

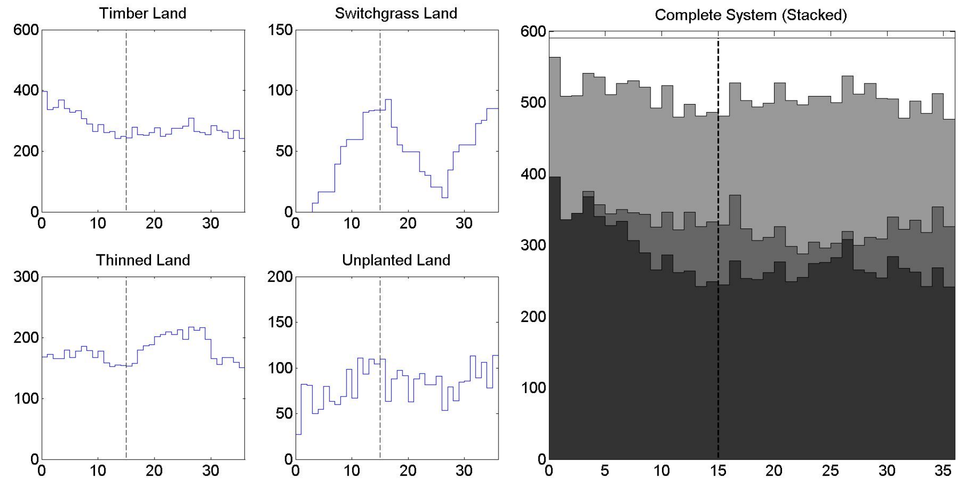

Figure 31: 15 Year Transition Timberlands Schedule (moderate land, harvester constrained initial state)

Also, the subplot of switchgrass type land shows a large decrease in usage around the middle of the transition period.

Figure 32 shows the unconstrained moderate land case with a 15 year transition period. The subplot of switchgrass land for this case does not yield the same single drop in land usage that the other biorefinery cases do (both 10 and 15 year cases). The fact that this case initially invested in a biorefinery leads us to believe that adding more time to the initial transition period allows for the harvesters to change to a more consistent management schedule.

None of the 20 year transition period scenario solutions were obtained. With an 8 hour long calculation time, GAMS was unable to converge on to solutions within 50% tolerance of the optimal solution. The problem size and complexity could yield a problem too difficult for CPLEX to find a solution. Alternative methods are needed to optimize these problems.

4.5 Conclusions

This chapter discussed how harvesting and planting schedules for a timberlands supply chain can be adapted to provide material to a new biofuel facility. An initial in depth

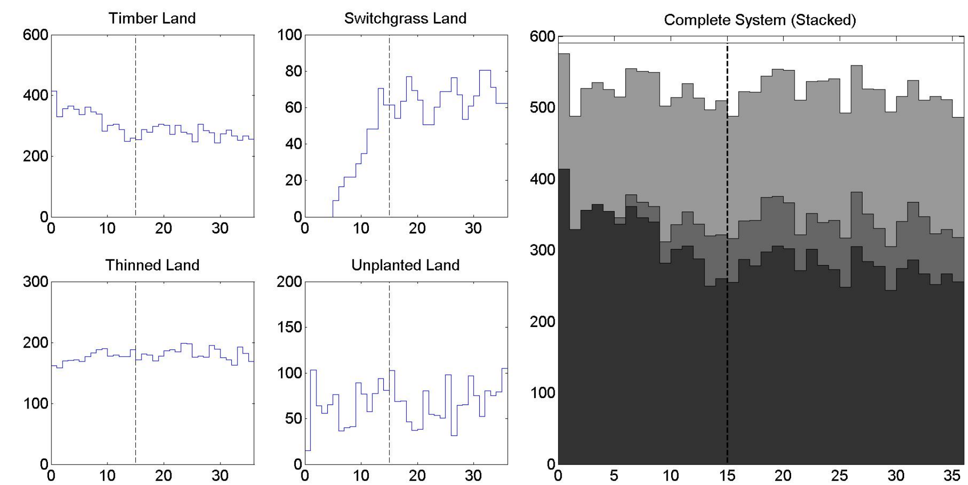

Figure 32: 15 Year Transition Timberlands Schedule (moderate land, unconstrained initial state)

study was performed on the generation of established harvesting cycles and initial states. We implemented smoothing constraints to promote consistency throughout the time schedule as well as generate different initial states for the transition state analysis. The behavior of systems with less land capacity was also studied. The 10 year transition period study led us to conclude that the decision to invest in biofuels had a complex relationship with the initial state. We also determined that with systems that previously did not invest in biofuels would change their decision with a longer transition period. The studies in this chapter provide an in depth initial look at the forestry land management problem from a mathematical programming perspective. Additional studies and further development on the model could yield a useful decision making tool for timberlands managers.

4.6 Cyclic Model Simulation Data

The parameter values for Chapter 4 are listed in this section.

Table 54: Mature Timber Biomass Densities (ton/acre) [$\rho_{kht-\tau}$]

	Age	18	19	20	21	22	23	24	25
timber	saw logs	64.97	69.65	79.94	83.20	92.18	95.66	99.13	102.60
	fiber logs	105.15	112.41	114.06	119.76	119.72	125.20	130.68	136.16
	residuals	56.71	60.69	64.67	67.65	70.63	73.62	76.60	79.59
thinned	saw logs	129.9336	139.2932	159.8776	166.3911	184.3697	191.3151	198.2605	205.206
	fiber logs	27.8429	29.84854	20.62937	21.46982	10.84527	11.25383	11.66238	12.07094
	residuals	32.75867	34.76431	36.76994	39.44412	42.11829	44.79247	47.46665	50.14082

Table 55: Young Timber Biomass Densities (ton/acre) [$\rho_{kht-\tau}$]

Age	8	9	10	11	12
residuals	121.48	159.92	198.89	247.75	294.95
thinnings	32.09	42.79	53.48	64.18	74.88
prunings	20.25	24.60	28.41	33.03	39.33

Table 55 is the amount of material obtained if young timber is harvested. The amount of material gained from thinning ($\hat{\rho}_{kht-\tau}$) is half of these values due to row harvesting.

The costs to harvest timber are \$11.00/acre for timber, \$5.00/acre for switchgrass, and \$7.00/acre for thinned land. The cost for thinning is \$6.00/acre. Yearly prices for saw logs and fiber logs were taken from [10]. The years for this reference range from 1976 to 2009. To fulfill the ranges for the simulation, the inflation numbers from [4] were taken from 1976 to 2014. The prices for missing years were generated by utilizing the inflation data. The first column of 58 shows a set of prices over 21 zones that were generated by our industrial source. Using this as a base, the prices from 58 were varied over the 21 harvesters. Prices to external buyers were 80% of the internal prices.

Table 56: Transportation Costs [y2010 $'s] [$s_{imt}$ and $\hat{s}_{iet}$]

	Manufacturer								Biorefinery Location		
Harvest Zone	1	2	3	4	5	6	7	8	Loc. 1	Loc. 2	Loc. 3
1	4.20	4.81	7.23	9.62	-	-	-	-	9.39	10.82	10.11
2	6.84	5.82	9.62	7.68	-	-	-	-	7.47	7.48	10.66
3	8.11	5.82	9.34	6.17	-	-	-	-	6.96	5.67	11.45
4	8.75	6.84	7.47	5.47	8.11	-	-	-	7.31	8.21	6.77
5	7.16	5.13	9.39	4.37	9.62	-	-	-	7.88	8.82	4.75
6	5.82	3.62	5.47	5.47	12.75	-	-	-	3.45	4.32	6.23
7	2.61	4.45	4.45	8.12	-	-	-	-	5.37	8.13	4.64
8	4.60	6.50	3.50	9.00	-	-	-	-	9.55	10.58	12.95
9	4.51	5.89	3.26	7.68	-	-	-	-	8.28	6.27	13.58
10	7.54	7.54	3.90	8.23	14.00	-	-	-	9.71	6.27	8.99
11	12.75	6.38	4.60	4.60	12.75	-	-	-	5.04	2.35	8.74
12	7.79	4.75	4.37	4.37	14.00	-	-	-	8.58	6.88	7.22
13	9.39	6.52	5.82	3.62	9.39	-	-	-	7.48	9.15	12.16
14	14.00	9.07	7.47	5.82	7.79	-	-	-	11.77	13.03	5.37
15	13.50	9.07	7.79	5.47	6.84	-	-	-	7.48	10.19	8.61
16	-	-	-	-	10.11	10.82	10.82	4.14	19.27	19.27	9.52
17	-	-	-	-	10.66	7.48	7.48	5.24	17.06	17.06	7.47
18	-	-	-	-	11.45	5.67	5.67	6.88	15.80	15.80	6.96
19	-	-	-	-	6.77	8.21	8.21	5.66	15.80	15.80	7.31
20	-	-	-	-	4.75	8.82	8.82	7.22	13.27	13.27	7.88
21	-	-	-	-	6.23	4.32	4.32	6.49	11.61	11.61	3.45

Table 57: Conversion Factors [α_{kl}]

Biomass Input	Product Lumber (ton/ton)	Gasoline (gallon/ton)	Pulp (ton/ton)	
Saw Logs	0.27	47.75	-	(wet)
Residuals	-	92.21	0.2	(dry)
Shavings	-	98.80	0.54	(dry)
Fiber Logs	-	47.75	0.27	(wet)
Prunings	-	85.62	0.25	(dry)
Grass	-	105.38	-	(dry)
Thinnings	-	85.62	0.25	(dry)

Table 58: Timberland Costs [Chapter 4]

	Timber Prices (y2010 \$/ton) [c_{ik}]	Planting Costs [$\bar{c}_{iht}$] (y2014 \$/acre) timber	switchgrass	Land Capacity (acre) [B_i]
1	\$31.62	70.00	46.00	125.81
2	\$33.20	75.00	45.00	36.47
3	\$33.20	70.00	45.00	23.56
4	\$29.16	65.00	50.00	18.68
5	\$29.16	70.00	50.00	21.66
6	\$29.16	70.00	46.00	38.63
7	\$25.23	75.00	46.00	89.22
8	\$25.23	75.00	46.00	29.87
9	\$25.23	60.00	46.00	58.95
10	\$19.13	70.00	45.00	34.94
11	\$19.13	70.00	46.00	22.63
12	\$24.69	70.00	50.00	16.54
13	\$24.69	70.00	51.00	54.06
14	\$24.69	65.00	45.00	16.06
15	\$24.69	80.00	50.00	12.40
16	\$28.20	70.00	46.00	16.27
17	\$28.84	70.00	50.00	18.44
18	\$28.84	65.00	50.00	24.43
19	\$29.05	60.00	51.00	30.68
20	\$30.32	80.00	50.00	18.68
21	\$30.32	60.00	46.00	16.26

Table 59: Biomass Costs (y2006 \$/ ton) [c_{ikt}]

	Prunings	Grass	Thinnings	Residuals
1	2.17	3.79	2.71	18.08
2	2.27	3.98	2.84	18.99
3	2.27	3.98	2.84	18.99
4	2.00	3.50	2.50	16.68
5	2.00	3.50	2.50	16.68
6	2.00	3.50	2.50	16.68
7	1.73	3.03	2.16	14.43
8	1.73	3.03	2.16	14.43
9	1.73	3.03	2.16	14.43
10	1.31	2.29	1.64	10.94
11	1.31	2.29	1.64	10.94
12	1.69	2.96	2.11	14.12
13	1.69	2.96	2.11	14.12
14	1.69	2.96	2.11	14.12
15	1.69	2.96	2.11	14.12
16	1.93	3.38	2.42	16.13
17	1.98	3.46	2.47	16.49
18	1.98	3.46	2.47	16.49
19	1.99	3.48	2.49	16.61
20	2.08	3.64	2.60	17.34
21	2.08	3.64	2.60	17.34

Table 60: Harvesting Costs (\$ per ton) [f_{iht} and $\hat{f}_{iht}$]

Timber	11
Switchgrass	5
Thinned Land	7
Thinning Costs	
Timber	6

Table 61: Manufacturer Data [Chapter 4]

Fac.	Product	Capacity (10^3 tons) $[G_m]$	Demand (10^3 tons) $[A_{ml}]$	Process Cost (y2014 $/ton) $[n_{mt}]$	Transport Cost to Pulp Mill (y2010 $/ton)	Product Price (y2010 $/ton) (Pulp: y2014 $/ton) $[p_{mlt}]$	Byproduct Price (k = Shavings) (y2010 $/ton) $[\hat{c}_{mkt}]$
1	Lumber	530	128.79	29.00	14.09	588.00	55.00
2	Lumber	520	126.36	28.00	12.90	504.00	55.00
3	Lumber	460	111.78	32.00	8.41	550.00	55.00
4	Lumber	460	111.78	29.00	6.21	588.00	50.00
5	Lumber	510	123.93	29.00	5.61	450.00	55.00
6	Lumber	320	77.76	33.00	5.65	462.00	60.00
7	Lumber	300	72.90	31.00	5.88	490.00	50.00
8	Pulp	2000	486.00	147.00	-	804.35	-

Table 62: Biorefinery Parameter Data (y2014 $)

	Capacity $[\hat{G}_u]$ [tons]	Product Price (/ton) $[p_{mlt}]$ Loc. 1	Loc. 2	Loc. 3	Processing Cost $[\hat{n}_{ut}]$ [/ton]	Capital Cost (millions) $[\beta_{ue}]$ Loc. 1	Loc. 2	Loc. 3
small	1750	479.00	489.00	490.00	73.20	1.234	1.198	1.284
medium	2500	480.00	482.00	477.00	67.00	1.548	1.518	1.582
large	3500	469.00	475.00	462.00	57.77	1.885	1.848	1.923

Table 63: Other Data

Planting Lower Bound $[R]$	25%
Harvesting Lower Bound $[H]$	10%
Log to Shavings Conversion $[\gamma_{kk'}]$	0.3

CHAPTER V

FUTURE WORK

Many different research ideas were considered during the course of this thesis, and many of these ideas were not pursued due to time constraints. This section discusses these ideas and any initial studies that were performed in the area.

5.1 Branching from the Single Level/Bilevel Comparison Study

After the single level/bilevel problem comparison studies, we pursued the idea of setting internal pricing of the bilevel model to encourage the same behavior as that in the single level model. These studies involved the area of inverse optimization [5]. The bilevel problem was reformulated where the original variables were now parameters input from the single level problem and the internal prices were now variables. With pricing as a variable, the model simplified to simply the KKT conditions. This was because the pricing only existed in the objective function and KKT constraints. The issue with this new formulation is that the KKT variables have no upper bound, yielding an unbounded problem. Inverse optimization techniques are usually applied to single level problems. Their use in a bilevel problem will need to be further explored.

Another subject that was discussed was utilizing trilevel models to represent the system. The trilevel model introduces a new, third level, which would be used to represent the biorefineries in the supply chain system. As the third level, the biorefinery would have the least priority for resources. Trilevel system implementation is much more rare than bilevel implementation, so studies will further prove their usefulness within supply chain models. Also, since these models are not commonly used, there are few solution methods. Many trilevel solution methodologies are modified bilevel algorithms [9]. In [81], a Kth-best algorithm is proposed to solve linear trilevel problems. The top level problem is solved using the simplex method, a common technique for finding solutions to constrained linear programming problems. The middle level is solved with the simplex method while setting the top level variable to an initial result. The Kth-best algorithm is used to determine whether the second result is an optimal solution of the two bottom levels. Finally, the lowest level is solved using simplex by holding the two other variables constant. These steps generate a set of extreme points for the problem until a global solution is found.

5.2 Branching from the Two Stage Multiperiod Bilevel Model

Now that initial structural studies have been completed for the bilevel models, the complexity of these models can be increased. A larger sized problem can be used (more decision makers, biomass types, manufacturer types), and new biorefinery decisions can be introduced, such as technology and facility integration. More realistic representations of pricing using supply-demand equilibrium can be used.

There are some groups currently researching more complex bilevel models for biofuel supply chains [79]; however, these models are much smaller in scale. We discovered in our study that the current algorithms struggle with solving large numbers of bilevel scenarios, so with an even more complex problem, new solution approaches will be needed. One such direction is to explore Bender's decomposition to solve the multiperiod problem as a whole, instead of using enumeration.

Another direction to explore is moving the problem to the dynamic realm. Unlike static games, dynamic games consider multiple plays of a game [27]. Repetition of a static game strategy may not necessarily yield the most optimal outcome. Timing becomes very important within these games. Analysis of a problem played many times yields optimal strategies much different than that of a static game. Players must weigh the tradeoffs of current decisions relative to future games. A dynamic representation can more realistically capture the idea of limited resources through a time period.

In dynamic programming, states are defined as games played at different stages in time [57]. Parameters in later states may change depending on the decisions from previous games.

The general form of a cost function in a dynamic game is as follows,

$$F(S,x) = \int_{t_0}^{t_f} C_t(S_t, x_t)\, dt. \tag{93}$$

where t represents time, S_t is the state variable, and x_t is the decision variable. Function $C_t(S_t, x_t)$ is the cost function of the system at each point in time. Both the state variable and the decision variable are functions of the time within the system.

$$S_{t+1} = S^M(S_t, x_t) \tag{94}$$

The state variable captures information about a state (in this case, a particular game at that time) that influences the decisions made by the player. Equation (94) shows that traveling to the next state is dependent on the current state and the decision that is made at that time. This state equation solely depends on the situation that is being modeled as well as what is being represented by the state variable S_t.

Dynamic games can be used to capture interactions between many players, making them a good choice to model multiple stage bilevel games. Furthermore, they are often used to study resource management problems [57]. Implementation of a dynamic form allows the analysis of the problem over specific time periods. The mathematics behind dynamic bilevel problems were first studied by Chen and Cruz [17].

The form of bilevel dynamic problems is similar to that of single decision maker systems, except the cost functions and constraints can now be influenced by the alternate player's decisions [76]. For example, the follower's objective function is

$$f(S, x, y) = \int_{t_0}^{t_f} c_t(S_t, x_t, y_t)\, dt \tag{95}$$

while the leader's objective function is

$$F(S, x, y) = \int_{t_0}^{t_f} C_t(S_t, x_t, y_t)\, dt \tag{96}$$

States are also modified as such

$$S_{t+1} = S^M(S_t, x_t, y_t) \tag{97}$$

The modification to the single player system is the inclusion of variable y_t, the follower's decision variable. Note that these equations are continuous over time, but sometimes the situation being modeled occurs in time periods. Thus, it must be formulated with discrete time.

The general form of the discrete time model is shown in equation (98).

$$F(S, x) = \sum_{t_0}^{t_f} C_t(S_t, x_t, y_t) \tag{98}$$

Instead of integrating over the time frame, the costs at each game are summed to determine the value of the overall system [51]. Despite having no integration, the games at each time point cannot be optimized as a static game. The influence of previous decisions must be understood before the choices can be determined at each time point. Earlier studies in this area focused on interaction of state information. In open loop systems, future states are only reliant on the initial state [49]. The players do not know the state information between game iterations. Therefore, players set their control law beforehand. Since information cannot be determined after play, the decision makers essentially set their strategy at the initial stage. While much simpler to solve, the flaw with this method is that it does not realistically describe most two player dynamic interactions [11].

In closed loop formulations, state information is available for decision makers after each move. Closed loop decisions are made according to the current game as well as state information from previous time periods. One issue with the closed loop form is that the principle of optimality for dynamic problems may not necessarily be satisfied due to the possible deviation from strategy by the leader. The principle of optimality is defined by the Bellman equation [57].

$$V_t(S_t) = \max_{x_t}(C_t(S_t, x_t) + V_{t+1}(S_{t+1})) \tag{99}$$

Function C_t is the cost function of the current game based on the state and decision variable. Function V_t is the value function of the state at time t. This equation is a recursive representation of dynamic games. Bellman's principle of optimality states that, no matter the initial decision and initial state, the remaining decisions must be optimal in relation to the next state, the state resulting from the initial game. Thus, if this policy applies to the problem, an optimal solution exists.

In a bilevel dynamic game under the Stackelberg closed loop framework, the follower has no incentive to deviate from the optimal strategy, since the follower tries to play optimally based on the leader's decision[28]. However, nothing prevents the leader from deviating through the time horizon. In a future game, the leader can inspect the remaining cost to go function and change strategy. The cost to go function captures the value of the remaining states within the game. This decision is made in spite of the previous states, thus, the player will change strategy to optimize the remaining play.

In literature examples, open and closed loop strategies are more often used to capture continuous systems [47]. Closed loop strategies can be modified to force optimal play as well as portray discrete state systems. This modified strategy is labeled as feedback strategy[28]. A constraint is added which chooses the optimal decision variable for the cost to go function.

For the first player, the constraint is:

$$x_t^* = \underset{x_t}{\operatorname{argmin}}(\sum_{t_0}^{t_f} C_t(S_t, x_t, y_t^*(x_t))) \tag{100}$$

and the second players constraint:

$$y_t^*(x_t) = \underset{y_t}{\operatorname{argmin}}(\sum_{t_0}^{t_f} c_t(S_t, x_t, y_t)) \tag{101}$$

As seen above, the leaders constraint is influenced by the optimal decision of the follower. Both constraints also take into account the value of the future states with function C_t. The most common method of solving dynamic bilevel problems is through recursion beginning at the final state. The calculations yield Riccati-type equations that can be solved to determine an optimal solution. An algorithm for obtaining these results for M-level problems is given by Gardner and Cruz [28]. Nie has also developed an algorithm that solves from the final state in a recursive manner [49, 50, 51]. However, to solve in this manner, final state information must be known.

Another approach to solving the dynamic bilevel problem could be to analyze solutions from the initial state, which would calculate potential pathing and final values. One of our first first ideas was utilizing an Approximate Dynamic Programming (ADP) methodology with bilevel problems. This can be done with Powell's approximate dynamic programming [57]. Unlike most dynamic solution algorithms, approximate dynamic programming steps forward in time. Stepping forward allows the exploration of branching paths. However, this runs into the issue of the three curses of dimensionality. These curses state that, since each node will branch into further paths, if the time horizon is fairly long, the number of possibilities will exponentially grow to become unmanageable. Powell solves this issue by suggesting repeated simulations across the problem. This determines average state values at each node, which can be used to determine the most optimal decision pathing.

With the bilevel model already being highly complex, the transition to dynamic programming will yield much more difficult calculation issues. Using ADP may make solving the problem more approachable.

5.3 Branching from the Cyclical Harvest Schedule Model

As we were developing the cyclic model, we discovered a wealth of ideas that could be explored. Unfortunately, due to time constraints, there were some directions we were unable to touch on.

The variable land study was briefly discussed in this document; we only utilized it during the initial state study. An alternate transition state model was programmed that allowed for the gradual deactivation of harvest lands through the time horizon. The constraint is set so that deactivated harvesters cannot reactivate. Therefore, the deactivation decision can only happen in the transition state because a deactivated harvester in the cyclic time period will not be able to satisfy the cyclic constraints. Unfortunately, for this initial test, the solver was unable to converge on a solution within reasonable time. One alternative that was considered was to limit the deactivation decisions to 5 year intervals, thereby reducing the complexity by removing a large number of binary decisions.

One limitation of the cyclic model was its single level form. As discovered in the single level/bilevel comparison study, in our single level problem, the internal prices of the biomass has no effect on the objective function. Therefore, for manufacturers, obtaining resources only incurred transportation and processing costs. This could yield decisions that would not be realistic in the real supply chain system. This limitation should be further explored.

While the generation of initial states yielded large amounts of results, we feel there is still a lot to explore. An important question is how the initial state effects the biorefinery investment decisions. Another set of case studies that were unable to be performed were studies on alternate cyclic constraints. Constraints 102 and 103 increase the flexibility of the system by ensuring that the total state of the timberlands are the same but does not require specificity to each harvester.

$$\sum_i S_{ih30\tau+20} \geq \sum_i (1-\omega) S_{ih10\tau} \ \forall \ h, \tau \tag{102}$$

$$\sum_i S_{ih30\tau+20} \leq \sum_i (1+\omega) S_{ih10\tau} \ \forall \ h, \tau \tag{103}$$

Equations 104 and 105 require the total amount of land planted in each harvester at the final state to be within a range of the total amount planted at time $t = 10$. This leads to similar amounts of land usage at both the initial and final states.

$$\sum_\tau S_{ih30\tau+20} \geq (1-\omega) \sum_\tau S_{ih10\tau} \ \forall \ i, h \tag{104}$$

$$\sum_\tau S_{ih30\tau+20} \leq (1+\omega) \sum_\tau S_{ih10\tau} \ \forall \ i, h \tag{105}$$

The final set of constraints (106 and 107) give more flexibility to the age distribution of the final state. As long as the distribution of material at a range of $\tau \pm \hat{t}$ is within the given tolerance, the cyclic conditions are met.

$$\sum_{\tau'=\tau-\hat{t}}^{\tau+\hat{t}} S_{ih30\tau'+20} \geq (1-\omega) \sum_{\tau'=\tau-\hat{t}}^{\tau+\hat{t}} S_{ih10\tau'} \ \forall \ i, h, \tau \tag{106}$$

$$\sum_{\tau'=\tau-\hat{t}}^{\tau+\hat{t}} S_{ih30\tau'+20} \leq (1+\omega) \sum_{\tau'=\tau-\hat{t}}^{\tau+\hat{t}} S_{ih10\tau'} \ \forall \ i, h, \tau \tag{107}$$

These 3 sets of constraints could yield new and interesting initial conditions as well as different solutions for the transition period studies. Given more flexibility, some of the systems that did not invest in biofuels may yield a different outcome.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND CONTRIBUTIONS

Three models were designed over the course of this thesis. These models utilized optimization and game theoretic frameworks to represent an established timberlands supply chain that was looking to invest in biofuel production.

The contribution from Chapter 2 was the development of a bilevel model representing an established timberlands supply chain with biorefinery interests. We explore the viability of Stackelberg games and bilevel models to represent supply chain interactions for separate but interconnected decision makers. This was shown by comparing the bilevel problem with a single level formulation. It was determined that certain factors, such as internal biomass pricing, would effect the behavior of the bilevel problem but not the single level one. Also, we discovered that in the bilevel problem, due to the competitive nature of the representation, harvesters and manufacturers would exhibit defensive behavior to ensure maximization of their own objective. In the single level problem, as long as the overall system's profit was optimal, individual manufacturers might operate at an internal loss. We concluded that the bilevel model was a useful formulation that yielded behaviors that a single level model could not represent. We believe that the results obtained from this competitive game theory application described more realistic behavior in the timberlands supply chain. A supply chain model with a more realistic representation would be more useful for decision making purposes.

The contribution of the second model (Chapter 3) was the combination and application of the two stage multiperiod representation with bilevel programming. We discovered that traditional solver algorithms had difficulty in solving problems with multiple bilevel scenarios; an enumeration method was used to optimize our models. The solution method studies also showed that selecting M values that are too small or large can cause difficulties in solving bilevel problems with the big M formulation. Our results showed that there are scenarios where biorefinery will be beneficial to a timberlands supply chain. The biorefineries in our study were reliant on timber byproducts for biofuel production, which created a revenue stream for timber harvesters and manufacturers from previously low value product that was typically discarded. The potential revenue from biofuel production as well as byproduct sales could drive the harvesters and manufacturers to increase productivity. We also performed a sensitivity analysis on the uncertain parameters to determine their impact on decision making. This information could be useful for timberlands managers and decision making. We also performed case studies on system activity by deactivating harvesters and manufacturers. These simulations are useful in showing how the system could be impacted by the loss of production in harvesting or manufacturing. The results from the two stage multiperiod revealed interesting insights for the timberlands system and showed that this type of representation could provide useful information for supply chain managers.

In Chapter 4, we ventured away from Stackelberg games to a modified discrete time cyclic scheduling model used to determine how timberlands planting and harvesting schedules should adapt to sustain a new biorefinery investment. The contributions of this chapter were not only an analysis on timberlands management over the long term, but the development of a modified cyclic scheduling model. This modified cyclic model allowed the system a transition period to adjust harvesting and planting schedules to coordinate with a new biorefinery. In the case studies, the transition periods were 10, 15, and 20 years in length. While, in our system, 15 and 20 year transition times were sufficient to adapt the lands for biofuel production, for the 10 year transition period studies, only 3 of the 6 scenarios yielded a biorefinery. We discovered that the system's ability to implement biofuel production was sensitive to the initial state of the forest. It was also discovered that, in situations where biofuel production was invested in, the system tried to shift to a more switchgrass oriented land build. Initial state generation was explored, and it was found that, through smoothing constraints, we could generate different initial states. We also determined that, for our system, a more land constrained problem yielded a much harder problem to solve. The results of this model would be incredibly useful for timberlands management, even those without biofuel interests. Managers looking to push for more potential profit could utilize this model to adapt their lands to a new harvest rotation. Through these studies, we realized the depth of research potential in this area, many ideas of which were discussed in the Future Works Section (5). This adapted cyclic schedule analysis is useful for planning in any supply chain with long term resource requirements.

The research presented involved the modeling of an established timberlands supply chain with biorefinery interests with mathematical programming approaches. This research involved new applications of the bilevel model framework to biorefinery investment planning in a timberlands system. The case studies discussed in this thesis has shown the usefulness of bilevel programming in supply chain optimization. Furthermore, a modified cyclic scheduling formulation was implemented in a timberlands biorefinery coordination problem, revealing the potential that this representation has for long term planning problems.

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